Iowa Research Online
The University of Iowa's Institutional Repository

University of Iowa
Iowa Research Online

Theses and Dissertations

Spring 2018

Essays in applied microeconomic theory

Charlène Lisa Cosandier
University of Iowa

Follow this and additional works at: https://ir.uiowa.edu/etd

Part of the Economics Commons

Copyright © 2018 Charlène Lisa Cosandier

This dissertation is available at Iowa Research Online: https://ir.uiowa.edu/etd/6084

Recommended Citation
Cosandier, Charlène Lisa. "Essays in applied microeconomic theory." PhD (Doctor of Philosophy) thesis, University of Iowa, 2018.
https://doi.org/10.17077/etd.ob3t5jyc

Follow this and additional works at: https://ir.uiowa.edu/etd
Part of the Economics Commons

المنارة للاستشارات

www.manaraa.com

ESSAYS IN APPLIED MICROECONOMIC THEORY

by

Charlène Lisa Cosandier

A thesis submitted in partial fulfillment of the
requirements for the Doctor of Philosophy
degree in Economics
in the Graduate College of
The University of Iowa

May 2018

Thesis supervisor: Professor Rabah Amir

www.manaraa.com

Graduate College
The University of Iowa
Iowa City, Iowa

CERTIFICATE OF APPROVAL

PH.D. THESIS

This is to certify that the Ph.D. thesis of

Charlène Lisa Cosandier

has been approved by the Examining Committee for the thesis requirement for the Doctor of Philosophy degree in Economics at the May 2018 graduation.

Thesis committee: ______________________________
Rabah Amir, Thesis Supervisor

____
Michael (Yu-Fai) Choi

____
Suyong Song

____
Anne Villamil

____
Nicholas Yannelis

المنارة للاستشارات
www.manaraa.com

ACKNOWLEDGEMENTS

I am indebted to my supervisor Rabah Amir from who I learned a lot. I also thank Anne Villamil, Nicholas Yannelis, Michael Choi and Suyong Song for their valuable comments and agreeing to serve on my committee. Many thanks to my mother and, of course, Thomas J. Rivera for their support.

www.manaraa.com

ABSTRACT

This thesis is composed of four independent essays in applied microeconomic theory. The first chapter examines the formation of bidding cartels in second-price sealed-bid almost-common value auctions. Cartel leaders discriminate between advantaged and regular bidders through their schedule of side-payments. If there is only one cartel, the cartel leader can attract regular bidders without leaving rents so that he acquires information about the common value at no cost. Advantaged bidders have an incentive to stay out due to their positive probability to win the auction. Nevertheless, we show than any stable cartel must include all regular bidders but fails to be all-inclusive. In the case of two competing cartels, equal-sharing of the asset value yields a stable cartel structure if both cartels are of same size. Finally, we find that a seller prefers a cartel structure with symmetric groups.

The second chapter provides a new approach to the basic issues of existence, uniqueness and comparative statics of Cournot equilibrium by using the properties of a fictitious objective function or an aggregate potential for symmetric Cournot oligopoly. Under this novel perspective, we are able to re-derive a number of existing results, as well as develop some general second-order properties for the equilibrium profit and social welfare functions with respect to the number of firms and the unit cost.

The third chapter investigates the effects of increased transparency on prices in the Bertrand duopoly model. Market transparency is defined as the proportion of

www.manaraa.com

consumers that are fully informed about the market and thus not captive to one firm. We consider two main cases of strategic interaction, prices as strategic complements and as strategic substitutes. For the former class of games, conventional wisdom concerning prices is confirmed, in that they decrease with market transparency. Consumer welfare always increases with higher transparency but changes in firms' profits are ambiguous. For the latter class of games, an increase in market transparency may lead to an increase in one of the prices, which implies ambiguous effects on both consumer welfare and firms' profits. An example with linear demand for differentiated products is also investigated. The results of the paper shed light on the mixed evidence concerning the effects of the Internet on retail markets and may illuminate some of the ongoing related public policy debates.

Finally, the fourth chapter examines the standard symmetric two-period R&D model with a deterministic one-way spillover structure: know-how flows only from the high R&D firm to the low R&D firm (but not vice-versa). Though firms are ex-ante identical, one obtains a unique asymmetric equilibrium (pair) in R&D investments, leading to inter-firm heterogeneity in the industry. The main part of the chapter provides a second-best welfare analysis in which we show that the joint lab yields a socially optimal R&D level subject to an equal treatment (of firms) constraint, which also coincides with the non-cooperative solution in the absence of spillovers. We also investigate the welfare costs of this equal treatment constraint and find that they can be quite significant.

المنارة للاستشارات
www.manaraa.com

PUBLIC ABSTRACT

The first chapter examines joint bidding in auctions with strong valuations asymmetries across bidders. With one cartel, the cartel leader can attract disadvantaged bidders and acquire information about the common value at no cost. With multiple competing cartels, cartel leaders also compete to attract regular bidders to become better informed at a low cost. A seller always prefers a large symmetric cartel structure.

The second chapter provides a new approach to the issues of existence, uniqueness and comparative statics of Cournot equilibrium by using the properties of a fictitious objective function. We rederive a number of existing results, and develop some general second-order properties for the equilibrium profit and social welfare functions with respect to the number of firms and the unit cost.

The third chapter investigates the effects of increased transparency on prices in the Bertrand duopoly model. Market transparency is defined as the proportion of consumers that are fully informed about the market. When prices are strategic complements, they decrease with market transparency. When prices are strategic substitutes, an increase in market transparency may lead to an increase in one of the prices, which implies ambiguous effects on both consumer welfare and firms' profits.

The fourth chapter examines a two-period R&D model where know-how flows only from the high R&D firm to the low R&D firm. Though firms are ex-ante identical, one obtains a unique asymmetric equilibrium in R&D investments, leading to heterogeneity in the industry. The second-best welfare analysis shows that the joint

المنارة للاستشارات
www.manaraa.com

lab yields a socially optimal R&D level subject to an equal treatment (of firms) constraint.

www.manaraa.com

TABLE OF CONTENTS

المنارة للاستشارات
www.manaraa.com

المنارة للاستشارات
www.manaraa.com

LIST OF FIGURES

المنارة للاستشارات
www.manaraa.com

CHAPTER 1

BIDDING CARTELS IN ALMOST COMMON-VALUE AUCTIONS

1.1 Introduction

The practice of joint bidding, whereby a group of bidders submits a common bid, is a widespread phenomenon that has brought much concern regarding their potential negative anti-competitive effects. Joint bidding, as opposed to collusion, is a legal practice and allowed in environments where the assets for sale require large investments, or which value is very uncertain. Merged bidders can then pool their information and/or financial resources together. However, such practices also raise competition policy concerns since they reduce the degree of competition and may therefore harm the seller, but also as they may foster collusive price-setting behavior. A classical example is that of auctions for offshore leases in 1975 where the eight largest crude oil producers ended up being banned from submitting common bids (see Hendricks and Porter, 1992).

Bidding consortia are common in very diverse areas: From procurement auctions to the sale of property rights in bankruptcy procedures, as well as in takeover battles through the formation of private-equity consortia. A common feature of these instances is the presence of asymmetries across bidders in terms of their valuations for the object or asset for sale. For example, Gorbenko and Malenko (2014) estimate that financial bidders (as opposed to strategic bidders) have higher valuations for mature

www.manaraa.com

poorly performing companies in takeover auctions. These valuations asymmetries are all the more exacerbated when some bidders hold a private-value advantage, in the sense that they persistently enjoy a higher ex-post valuation as compared to other bidders. A classical example is that of takeover battles in which some of the potential acquirers of the target hold a toehold, which spurs the aggressiveness of their bids (see Bulow et al., 1999; Singh, 2015).

In this paper, we examine the formation of bidding consortia in auctions with strong valuation asymmetries across bidders, namely, *almost* common-value auctions. We develop a two-stage model in which players first enter a cartel formation game which results in a partition of the (original) set of players. Cartels then compete in a second-price sealed-bid auction. A main feature of the model is to consider two types of bidders, advantaged and regular, in the sense that advantaged bidders have a higher ex-post valuation for the object. We assume that bidders' identity is common knowledge, in the sense that bidders' types are perfectly observable by every player. This is akin to a transparent bidding registration procedure in the terminology of Marshall and Marx (2009).

Most existing literature on joint bidding takes a mechanism design approach whereby an uninterested third party (the ring center) designs a mechanism, which consists of an allocation rule to represent the ring at the auction, together with an incentive feasible schedule of side-payments (McAfee and McMillan, 1992; Graham and Marshall, 1987; Mailath and Zemsky, 1991). As argued by McAfee and McMillan (1992), they mainly focus on independent private value settings since an ex-post

www.manaraa.com

efficient allocation rule is trivial in pure common-value settings as bidders have the same ex-post valuations by definition. Straightforwardly, given that the identity of bidders is common knowledge, the same observation applies here.

Thus, we suppose instead that bidders themselves may offer their peers shares of the asset value in exchange of cartel membership. Cartel leaders discriminate between advantaged and regular bidders through their schedule of sharing rules since they greatly differ in their ability to win the auction on their own, that is, when staying out of a cartel. In particular, we show that if there is only one cartel (possibly not all-inclusive), the cartel leader can attract regular bidders without leaving rents so that he acquires information about the common value at no cost.

Cho et al. (2002) instead endogenize the (pre-auction) process of coalition formation in the context of a first-price auction with perfectly known pure common value. They suppose that bidders are budget constrained and cannot acquire the object when bidding on their own. Thus, the formation of a coalition allows bidders to pool their resources. The present paper does not consider budget-constrained bidders. Rather, the main advantage of cartel formation here is to acquire additional information by pooling members' signals and to reduce competition in the auction. Recently, Troyan (2017) studies the optimal design of bribes in a two-bidder second-price auction with interdependent valuations. He shows that offering a bribing contract to one's competitor signals the proposer's private information about the object value and may lead to failed collusion.

Krishna and Morgan (1997) and Mares and Shor (2012) examine the effect

المنارة للاستشارات

www.manaraa.com

of joint bidding in pure common-value environments on the seller's expected equilibrium revenue. They focus on the special case where multiple cartels of identical size compete in a second-price auction but do not endogenize cartel formation. They identify two main effects - information pooling and inference effects- whereby both the bidding cartel and outside bidders benefit from a reduction in their winner's curse since the cartel has a more accurate estimate of the object value. This ultimately leads to more aggressive bids, which is beneficial for the seller's revenue. However, the decrease in competition and its negative impact on the seller's revenue is shown to dominate in the average value model for instance.

We first show that, if there is a unique advantaged bidder in the auction, then a cartel is stable (in the sense of d'Aspremont et al., 1983) if it is all-inclusive and led by the advantaged bidder. In particular, the cartel leader appropriates all rents and is perfectly informed about the common value. With more than one advantaged bidder, an all-inclusive cartel is no longer stable since advantaged bidders have an incentive to stay out due to their positive probability to win the auction. Nevertheless, we show than any stable cartel must include all regular bidders.

We then consider the case of two competing cartels. We find that, with equal sharing of the asset value, the cartel structure is stable if cartels are of same size. Likewise, the seller is better off when information is distributed symmetrically across groups.

The rest of the paper is organized as follows. Section 1.2 lays out the model. Equilibrium bidding behavior is analyzed in Section 1.3. Section 1.4 examines the

المنارة للاستشارات
www.manaraa.com

cartel formation game and Section 1.5 studies the case of competing cartels. Finally, Section 1.6 concludes.

1.2 Model setup

We consider the sale of one unit of an indivisible good through a second-price sealed-bid auction where joint bidding is allowed. The set of risk-neutral players is $N = \{1, 2, ..., n\}$ and we assume that each player $i \in N$ can be of two types: either *advantaged* ($\theta = a$) or *regular* ($\theta = r$) in the sense that advantaged bidders have a strictly higher ex-post valuation for the object. We assume that players' identity is common knowledge in the sense that types are perfectly observable[1], and we let $\mathcal{A}$ denote the set of advantaged players. Each player privately receives a signal x_i about the value of the object which has the following familiar additive form (Klemperer, 1998):

$$v^{\theta}(\boldsymbol{x}) = \alpha(\theta) \sum_{i=1}^{n} x_i \quad \text{where } \alpha(\theta) = \begin{cases} 1 & \text{if } \theta = r \\ \alpha & \text{if } \theta = a \end{cases}$$

with $\alpha > 1$. Signals $X_i's$ are independently and identically distributed according to a continuous distribution F over $[0, 1]$, with $F' = f$ and $f(x) > 0$ for all $x \in [0, 1]$. Throughout, we assume that F is strictly log-concave.

Cartel formation stage. After learning their signals, players may propose their peers to form a cartel prior to the auction. The proposing bidder offers a schedule

[1]This may be the case if the bidding registration procedure is transparent (see Marshall and Marx, 2009).

المنارة للاستشارات
www.manaraa.com

of sharing rules of the asset value $\{(\lambda^a(x_i), \lambda^r(x_i))\}_{x_i\in[0,1]}$ such that $\lambda^\theta(x_i) \in [0,1]$ for all $\theta \in \{a, r\}$, $x_i \in [0,1]$, and other players simultaneously decide whether to join. If they accept, they restrain from bidding in the auction. The resulting set of active bidders at the auction is then a partition π of N. The proposing bidder becomes the cartel leader and gets to actively participate in the second stage. If the cartel wins the auction, then each member obtains his share of the common value. Throughout, we focus on ex-post efficient cartels so that a cartel leader is necessarily an advantaged player.

1.3 Joint bidding with heterogeneous bidders

In this section, we characterize equilibrium profiles of strategies of the auction stage assuming that the set of bidders consists of a cartel containing at least two members and (possibly) multiple outside bidders. This allows us to derive outside bidders' expected payoffs, which will be useful when analyzing players' participation decisions in the cartel formation game.

1.3.1 The case of a sole outside bidder

For the reader's convenience, we start our analysis by examining equilibrium bidding behavior in the special case where stage 1 resulted in the formation of a cartel of size $n-1$, so that only one player decided to stay out (i.e., $\pi = \{\mathcal{C}, \{h\}\}$, with $h \in N\backslash\mathcal{C}$ and $|\mathcal{C}| = n-1$): Namely, there are only *two* bidders in the auction. Throughout, we assume that there are at least two advantaged players. Since our focus is on efficient outcomes of the cartel formation game, it follows that the cartel

المنارة للاستشارات

www.manaraa.com

leader must in fact be an advantaged player. It is well-known that, in second-price *almost* common-value auctions with two bidders, the advantaged bidder always wins and the seller's revenue substantially decreases (Bikhchandani, 1988). In our context, if the outside bidder is regular, asymmetries across bidders are then exacerbated since the cartel leader (that is, an advantaged player) now also holds an information advantage over his competitor due to the cartel information pooling. Not surprisingly, this well-known result then carries over when the value-advantaged bidder also enjoys greater private information about the common value of the object.

Lemma 1.1. *If a regular bidder stays out of the cartel, then he always loses the auction.*

Proof. We first show that the regular bidder does not always win the auction. Suppose he receives a signal $x = 0$, then he bids at most $n-1$ as any higher bid is weakly dominated. In turn, if the cartel aggregate signal is $z = n-1$, in which case each cartel member's signal equals one, then it bids no less than $\alpha(n-1) > n-1$. Thus, there is no undominated equilibrium in which the outside regular bidder always wins. Therefore, if he were to win the auction with positive probability, then there must be an interval of bids that is in the range of both bid functions by assumption of continuity. Given the additive form of the object value, the arithmetic mean of the cartel's composite signal $(x_i)_{i \in \mathcal{C}}$ is also a sufficient statistic effectively summarizing its private information. Thus, let $\hat{x}$ and $\frac{\hat{z}}{n-1}$ be signals such that $b = \beta^o(\hat{x}) = \beta^c(\frac{\hat{z}}{n-1})$, where the cartel's aggregate signal is scaled by the factor $\frac{1}{n-1}$ so that both bid functions have the same domain. Suppose that $\alpha\,(\hat{z} + \hat{x}) > b$, then the cartel

www.manaraa.com

has a strict incentive to deviate to some $b' > b$ as winning the auction is strictly profitable. Instead, if $\alpha(\hat{z} + \hat{x}) < b$, then the cartel is better off slightly lowering its bid since winning yields a negative payoff. Thus, it must be that $b = \alpha(\hat{z} + \hat{x})$. A similar argument for the outside regular bidder shows that $b = \hat{z} + \hat{x}$ at equilibrium, which contradicts $\alpha > 1$. Hence, an outside regular bidder always loses the auction in undominated equilibrium. $\square$

Next, we turn to the case where the outside bidder is instead advantaged, so that both bidders (i.e., the cartel leader and the outside bidder) have identical ex-post valuations for the object. The only remaining asymmetry lies in the information structure since the cartel leader's information about v is much more accurate: The cartel leader holds a share of $\frac{n-1}{n}$ of the relevant information. Observe that the technical issues associated with multidimensional signals are alleviated here due to the additivity of the value function (see Goeree et al., 2003). The cartel's composite signal $\boldsymbol{s} = (x_i)_{i \in \mathcal{C}}$ is indeed effectively summarized by the (one-dimensional) aggregate signal $z = \sum_{i \in \mathcal{C}} x_i$. Clearly, the arithmetic mean of the cartel members' signals $\frac{1}{n-1} \sum_{i \in \mathcal{C}} x_i$ is an alternative sufficient statistic. This allows us to characterize a simple equilibrium profile of strategies with the familiar form of the standard symmetric case despite asymmetries across bidders (Mares, 2000)[2]. This equilibrium has indeed the desirable property that, if no cartel is formed (so that $\pi = \{\{1\}, \{2\}, ..., \{n\}\}$), it reduces to the symmetric equilibrium derived by Milgrom and Weber (1982).

[2] This is also the equilibrium selected by Troyan (2017) when studying the competition between a cartel and an outside bidder.

المنارة للاستشارات

www.manaraa.com

Proposition 1.2. *Suppose that advantaged bidder $h \in \mathcal{A}$ stays out and competes against a cartel of size $|\mathcal{C}| = n-1$ in the auction. The following strategies constitute an undominated (ex-post) equilibrium*

$$\beta^c(\boldsymbol{x}_{-h}) = \alpha \frac{n}{n-1} \sum_{i \neq h} x_i \quad \textit{and} \quad \beta^o(x_h) = \alpha n x_h$$

Proof. Let $z \equiv \sum_{i \neq h} x_i$ denote the cartel's (realized) aggregate signal. We now establish that the proposed strategies constitute an ex-post equilibrium. By bidding $\beta^c(z)$, the cartel wins if $\frac{z}{n-1} \geq x_h$, in which case it gets $\alpha\left[z - (n-1)x_h\right] \geq 0$, which ensures that it does not regret winning. Since the price it pays upon winning is the second highest bid, increasing its bid does not improve its payoff, while a lower bid triggers losing the auction resulting in a zero ex-post payoff. Likewise, the outside bidder does not regret losing since outbidding the cartel would lead to $\alpha\left[x_h - \frac{z}{n-1}\right] \leq 0$ by assumption. Similar arguments show that neither bidder suffers from ex-post regret in the case where $\frac{z}{n-1} \leq x_h$. Finally, note that $\beta^c(z) = \alpha \frac{n}{n-1} z > \alpha z = v^a(\boldsymbol{x}_{-h}, 0)$ and $\beta^o(x_h) = \alpha n x_h > \alpha x_h = v^a(\mathbf{0}, x_h)$, which ensures that the equilibrium strategies are undominated. $\square$

As pointed out by DeBrock and Smith (1983), the cartel's ability to pool information enhances the accuracy of its information about the common value, which in turn alleviates the cartel leader's exposure to the winner's curse (*information pooling* effect). Similarly, the outside bidder also benefits from competing against a better informed bidder since he infers that the cartel's bid is closer to the *true* value of the object, which in turn alleviates the winner's curse effect as well (*inference* effect).

www.manaraa.com

These two effects induce both players to bid more aggressively as compared to the competitive case where all n bidders submit individual bids (Krishna and Morgan, 1997).

Clearly, more aggressive bids have a direct positive impact on the seller's expected equilibrium revenue, but at the cost of reduced competition in the auction, which is instead harmful for revenue considerations. Mares (2001) examines which of these two countervailing effects dominates when two asymmetric cartels (i.e. of different sizes) compete in a second-price common-value auction. He shows that the seller's revenue is lower when the set of bidders is partitioned in two cartels of asymmetric sizes. In other words, the (anti-) competition effect dominates the inference and information pooling effects.

As the next result illustrates, the addition of a cartel member may nevertheless induce the cartel leader to bid more cautiously if its new piece of information (that is, the signal of the additional cartel member) is pessimistic about the true value.

Proposition 1.3. *The addition of a cartel member with realized signal x_{n+1} has the following effects:*

(i) The outside bidder always bids more aggressively

(ii) The cartel bids more aggressively if his additional information component is optimistic enough about the common value, i.e. for $x_{n+1} \geq \frac{1}{n^2-1} \sum_{i \in N \setminus \{h, n+1\}} x_i$

Proof. Let β_k^c and β_k^o respectively denote the equilibrium bidding strategies of a cartel and an outside bidder when $|\mathcal{C}| = k$.

(i) For the outside bidder, $\beta_n^o(x_h) - \beta_{n-1}^o(x_h) = \alpha x_h \geq 0$

المنارة للاستشارات

www.manaraa.com

(ii) For the cartel, we have that

$$\beta_n^o(z+x_{n+1}) - \beta_{n-1}^o(z) = \alpha\left[\frac{n+1}{n}(z+x_{n+1}) - \frac{n}{n-1}z\right] \geq 0$$
$$\Leftrightarrow x_{n+1} \geq \left(\frac{n^2}{n^2-1}-1\right)z = \frac{1}{n^2-1}\sum_{i\in N\setminus\{h,n+1\}} x_i$$

□

Not surprisingly, the larger the original set of players, the more aggressive the cartel's bid *even* if the additional signal is low. Intuitively, the cartel already holds a substantial share of information about the common value so that receiving an additional signal has a lower impact on its estimate of the object value.

1.3.2 The general case

We now examine the auction stage in which a cartel of size k competes against multiple outside bidders, with $2 \leq k \leq n-2$. Throughout, we focus on equilibrium profiles of strategies involving the use of *symmetric* bids among outside bidders, and shall refer to such as symmetric equilibria. For completeness, we first define two familiar functions $m(x) = x - \mathbb{E}[X|X \leq x]$ and $\bar{m}_k(x) = x - \mathbb{E}\left[\bar{X}_k|\bar{X}_k \leq x\right]$, and provide an intermediate result that will prove useful in the sequel.

Lemma 1.4. *Suppose that F is strictly log-concave. Then, for all $x \in [0,1]$,*

(i) *$m(x)$ and $\bar{m}_k(x)$ are strictly increasing in x.*

(ii) *$m(x) \geq \bar{m}_k(x)$ for all $k > 1$.*

For a proof of the first part, see for instance Bikhchandani and Riley (1991), while the proof of the second part is provided in Mares and Shor (2008). Next, we

المنارة للاستشارات
www.manaraa.com

closely follow the approach of Hernando-Veciana (2004) by proposing a function ϕ that maps signals of outside bidders into average signals of the cartel who submit the same bid in the (unique) equilibrium of the auction stage. More specifically, such function is implicitely defined by

$$\mathbb{E}\left[V|\bar{X}_k = \phi(s), Y_{1:(n-k)} = s\right] = \mathbb{E}\left[V|\bar{X}_k \leq \phi(s), Y_{1:(n-k)} = Y_{2:(n-k)} = s\right] \qquad (1.1)$$

This equation can be understood as follows: When an outside bidder wins the auction, the price he pays may be coming from the cartel or another outside bidder. Given the asymmetry of information between the cartel and outside bidders, outbidding the cartel allows the winner to get a more accurate estimate of the object value since it privately knows k signals. Clearly, the outside bidder's expected value for the object conditioning on the event where the second highest bid is coming from the cartel differs from his expected value given that the second highest bid comes from another outside bidder. The function ϕ as defined in Eq.(1.1) then simply provides the value of the cartel's average signal as a function of the winner's realized signal such that these two conditional expectations are equal.

Lemma 1.5. *There exists a unique function $\phi : [0,1] \to [0,1]$ implicitely defined by Eq.(1.1). Furthermore, ϕ is continuous, strictly increasing, and satisfies $\phi(0) = 0$ and $\phi(1) < 1$.*

Proof. We have

$$\mathbb{E}\left[V|\bar{X}_k = \phi(s), Y_{1:(n-k)} = s\right] = k\phi(s) + m(s) + (n-k)\mathbb{E}\left(X_i|X_i \leq s\right)$$

www.manaraa.com

and

$$\mathbb{E}\left[V|\bar{X}_k \leq \phi(s), Y_{1:(n-k)} = Y_{2:(n-k)} = s\right] =$$

$$2m(s) + (n-k)\mathbb{E}\left(X_i|X_i \leq s\right) + k\mathbb{E}\left(\bar{X}_k|\bar{X}_k \leq \phi(s)\right)$$

Equalizing these two and solving for $\phi(s)$ simply yields

$$k\left[\phi(s) - \mathbb{E}\left(\bar{X}_k|\bar{X}_k \leq \phi(s)\right)\right] = m(s) \Leftrightarrow \bar{m}_k(\phi(s)) = \frac{m(s)}{k} \Leftrightarrow \phi(s) = \bar{m}_k^{-1}\left(\frac{m(s)}{k}\right)$$

The uniqueness, continuity and monotonicity of ϕ directly follow from the fact that m and $\bar{m}_k$ are continuous and strictly increasing. Finally, we have that $\phi(0) = \bar{m}_k^{-1}\left(\frac{m(0)}{k}\right) = \bar{m}_k^{-1}(0) = 0$ and $\phi(1) = \bar{m}_k^{-1}\left(\frac{m(1)}{k}\right) < \bar{m}_k^{-1}(m(1)) = \bar{m}_k^{-1}(\bar{m}_k(1)) = 1$. □

We may now provide the main result of this section.

Proposition 1.6. *Suppose that the set of bidders is partitioned as $\pi = \{\{1, ..., k\}, \{k+1\}, ..., \{n\}\}$ with $2 \leq k < n$. There is a unique non-degenerate symmetric undominated equilibrium where the cartel bids*

$$\beta^c(x) = \begin{cases} \mathbb{E}\left[V|\bar{X}_k = x, Y_{1:(n-k)} = \phi^{-1}(x)\right] & \text{for all } x \in [0, \phi(1)] \\ \mathbb{E}\left[V|\bar{X}_k = x, Y_{1:(n-k)} = 1\right] & \text{for all } x \in (\phi(1), 1] \end{cases}$$

while outside bidders bid

$$\beta^o(x) = \mathbb{E}\left[V|\bar{X}_k = \phi(x), Y_{1:(n-k)} = x\right] \quad \text{for all } x \in [0, 1]$$

Proof. We first show that the proposed strategies form an equilibrium. Throughout, we let $\omega(x, y) \hat{=} \mathbb{E}\left[V|\bar{X}_k = x, Y_{1:(n-k)} = y\right]$. Suppose that outside bidders follow β^o and

المنارة للاستشارات

www.manaraa.com

that the cartel's average composite signal is t. Consider the case where $t \in [0, \phi(1)]$. By bidding b, the cartel wins whenever $Y_{1:(n-k)} \leq \beta^{o^{-1}}(b))$ so that its expected payoff is

$$\begin{aligned} U^c(b,t) &= \mathbb{E}\left[\left(V - \beta^o(Y_{1:(n-k)})\right)\mathbb{I}\{Y_{1:(n-k)} \leq \beta^{o^{-1}}(b)\}|\bar{X}_k = t\right] \\ &= \int_0^{\beta^{o^{-1}}(b)} \left(\omega(t,y) - \omega(\phi(y),y)\right) \mathrm{d}F_{1:(n-k)}(y|\bar{X}_k = t) \end{aligned}$$

Given the functional form of the common value V, ω is strictly increasing in each argument. Thus, for all $y < \phi^{-1}(t)$, $\omega(t,y) - \omega(\phi(y),y) > 0$, while $\omega(t,y) - \omega(\phi(y),y) < 0$ for all $y > \phi^{-1}(t)$. Hence, $b = \beta^c(t) = \omega(t, \phi^{-1}(t))$ is indeed optimal. Consider now the case where $t \in (\phi(1), 1]$ and observe that

$$\beta^c(t) = \omega(t,1) > \omega(\phi(1),1) \geq \omega(\phi(t),t) = \beta^o(t)$$

so that the cartel always wins when playing according to β^c, whatever outside bidders' signals. Thus, it suffices to show that the cartel does not have any profitable downward deviation (upward deviations clearly do not affect his payoff). Upon winning, the cartel gets

$$\omega(t,y) - \omega(\phi(y),y) > \omega(\phi(1),y) - \omega(\phi(y),y) \geq 0 \quad \forall y$$

while lowering its bid to some $b < \beta^o(1)$ may result in losing the auction with strictly positive probability, leading to zero expected payoff. Similar arguments show that playing according to β^o constitutes a best response for outside bidders, so that the proposed profile of strategies is indeed a non-degenerate undominated equilibrium.

Finally, we show that there is no other non-degenerate equilibrium. Suppose that b_c and b_o are two strictly increasing and differentiable functions that constitute a

المنارة للاستشارات
www.manaraa.com

non-degenerate equilibrium of the auction stage. The corresponding inverse bid functions, $\gamma_c(b)$ and $\gamma_o(b)$, are well-defined, increasing and continuous. Since all bidders have a positive ex-ante probability to win the auction by assumption, the intersection $(b_c(0), b_c(1)) \cap (b_o(0), b_o(1))$ is non-empty. For any $b \in (b_c(0), b_c(1)) \cap (b_o(0), b_o(1))$, it must be that $\gamma_c(b) = \phi(\gamma_o(b))$ from the analysis above and the definition of ϕ. Letting $\underline{b} \equiv \inf\{(b_c(0), b_c(1)) \cap (b_o(0), b_o(1))\}$ and $\bar{b} \equiv \sup\{(b_c(0), b_c(1)) \cap (b_o(0), b_o(1))\}$, we also have that $\gamma_c(\underline{b}) = \phi(\gamma_o(\underline{b}))$ and $\gamma_c(\bar{b}) = \phi(\gamma_o(\bar{b}))$ by continuity. Since $\phi(0) = 0$ and $\phi(1) < 1$, $\underline{b} = b_o(0) = b_c(0)$ and $\bar{b} = b_o(1)$. Hence, $b_c(x) = \beta^c(x)$ for any $x \in [0, \phi(1)]$ and $b_o(x) = \beta^o(x)$ for all $x \in [0, 1]$. Finally, we show that $b_c(x) = \omega(x, 1)$ whenever $x \in (\phi(1), 1]$. In such a case, any higher bid is weakly dominated by $\omega(x, 1)$, while a lower bid entails a positive probability of losing the auction and getting zero payoff. $\square$

1.4 Cartel formation

Given the equilibrium at the auction stage, we now examine the cartel formation game. We use the notion of cartel stability *à la* d'Aspremont et al. (1983) which ensures that no cartel member has an incentive to leave the cartel (*internal* stability) and no individual (outside) bidder has an incentive to join the cartel (*external* stability). In what follows, we let $F^c_\theta(k, x)$ denote the expected payoff of a type θ member of cartel $\mathcal{C}$ of size k with signal x, and $F^o_\theta(k, x)$ the expected payoff of a type θ outside bidder with signal x when the cartel is of size k, with $\theta \in \{a, r\}$.

Definition 1.1. Let π be a partition of N. Cartel $\mathcal{C} \in \pi$, with $|C| = k$, is stable if it

المنارة للاستشارات
www.manaraa.com

satisfies the following for all $\theta \in \{a, r\}$:

(i) Internal stability: $F^c_\theta(k, x) \geq F^o_\theta(k-1, x)$

(ii) External stability: $F^c_\theta(k+1, x) < F^o_\theta(k, x)$

Note that the inequality is strict for the external stability condition since we assume that, whenever indifferent, a player joins the cartel.

As stated above, we focus on the design of sharing rules that both induce truthful reports of private information, so that the cartel's composite signal yields an accurate estimate of the common value, and lead to a stable cartel. In this respect, observe that the internal stability condition is closely related to players' participation constraints when deciding whether to join the cartel. In sum, the internal stability condition can be interpreted as the participation constraint *evaluated at* the cartel leader's chosen sharing rule.

Since the cartel leader has only one "contracting" variable , i.e. the sharing rule, it directly follows that an incentive compatible schedule of sharing rules pools signals for each type of bidder (advantaged and regular), that is $\lambda^\theta(x_i) = \lambda^\theta(x_j)$ for any $(x_i, x_j) \in [0,1]^2$, $\theta \in \{a, r\}$. Since bidders' identity is common knowledge, the cartel leader may nonetheless discriminate between regular and advantaged bidders by proposing *type*-dependent sharing rules.

1.4.1 The case of an all-mighty advantaged bidder

To begin with, we focus on the simple case where there is only one advantaged player. We first show that even if regular players form a cartel and therefore enjoy an

المنارة للاستشارات
www.manaraa.com

information advantage, the cartel still cannot outbid the outside (value-) advantaged bidder. The idea here is that the inference effect previously described is magnified by the outside bidder's value advantage.

Lemma 1.7. *A cartel only composed of regular players never wins the auction.*

Proof. The proof is similar to that of lemma 1.1. Suppose the cartel's aggregate signal is $z = 0$ (so that each member's signal is zero), then it bids at most one as any higher bid is weakly dominated. In turn, if the outside advantaged bidder's signal is 1, then he bids no less than $\alpha > 1$. Thus, there is no undominated equilibrium in which the cartel always wins. Therefore, if it were to win the auction with positive probability, then there must be an interval of bids that is in the range of both bid functions by assumption of continuity. Let $\hat{x}$ and $\frac{\hat{z}}{n-1}$ be signals such that $b = \beta^o(\hat{x}) = \beta^c(\frac{\hat{z}}{n-1})$. Suppose that $\alpha\,(\hat{z} + \hat{x}) > b$, then the outside bidder has an incentive to slightly increase his bid as winning the auction is strictly profitable. Instead, if $\alpha\,(\hat{z} + \hat{x}) < b$, then he is better off slightly lowering his bid since winning yields a negative payoff. Thus, it must be that $b = \alpha(\hat{z} + \hat{x})$. A similar argument for the cartel shows that $b = \hat{z} + \hat{x}$ at equilibrium, which contradicts $\alpha > 1$. Hence, if the advantaged bidder stays out, a cartel led by a regular bidder never wins the auction. $\square$

Such a cartel is stable if the outside bidder's value advantage satisfies $\alpha \geq \frac{n+1}{n}$: his expected outside payoff by individually competing against the cartel in the auction outweighs the cartel's expected value for the object and, a fortiori, the maximum share of the value he could receive by becoming a member.

المنارة للاستشارات

www.manaraa.com

Instead, the advantaged player can form an all-inclusive cartel (i.e. containing all bidders) at no cost, namely, without any monetary transfers to regular players. Indeed, since an outside regular bidder always loses when competing against a cartel led by an advantaged bidder (see lemma 1.1), he will be indifferent between participating and staying out since he gets zero payoff in either case. Participation and incentive compatibility constraints of regular players are therefore trivially satisfied, so that the cartel leader becomes perfectly informed about the object value and appropriates the whole surplus. Such an outcome is straightforwardly ex-post efficient since the object is allocated to the bidder who values it the most - the cartel leader. In fact, our next result states that this is a stable cartel configuration when there is only one advantaged bidder.

Proposition 1.8. *Suppose that $|\mathcal{A}| = 1$. If a cartel is led by the advantaged bidder and all-inclusive (i.e. $\mathcal{C} = N$), then it is stable. Furthermore, the cartel leader appropriates the whole surplus: he leaves no rents to its members and the selling price is zero.*

Proof. Suppose that the cartel is led by the (unique) advantaged bidder and all-inclusive. Since it is externally stable by definition, it suffices to show that it also satisfies the internal stability property. To this end, observe that since an outside regular bidder never wins the auction when competing against a cartel led by an advantaged bidder (see lemma 1.1), regular players' participation and incentive compatibility constraints simply write

$$\lambda_i(x_i) \geq 0 \quad \forall x_i \in [0,1],\ i \in N \backslash \mathcal{A} \tag{PC}$$

www.manaraa.com

$$\lambda_i(x_i) \geq \lambda_i(x_i') \quad \forall (x_i, x_i') \in [0,1]^2, \ i \in N \backslash \mathcal{A} \tag{IC}$$

It directly follows that the leader's profit-maximizing incentive feasible schedule of sharing rules bunches signals and simply consists of a nill share, i.e. $\lambda^* = 0$. Hence, a regular player is indifferent between joining and staying out of the cartel, in which case he decides to join nonetheless by assumption[3]. Since this holds for any regular player, the cartel is also internally stable. □

Observe that regular players only have weak incentives to join the cartel (as they get zero utility in both cases). This result also suggests that ex-post efficient joint bidding is sustainable under both a weak and a strong cartel - that is both without and with side-payments (McAfee and McMillan, 1992).

1.4.2 Pooling of information versus rent erosion

We now go back to the general case with strictly more than one advantaged bidder. The cartel leader now faces a trade-off between attracting bidders to benefit from competition reduction at the auction stage, and rent sharing. Indeed, since advantaged bidders' outside expected payoffs are positive due to their positive probability to win the auction on their own, the cartel leader needs to offer positive shares to advantaged players in order to attract them and ensure cartel stability.

Since the cartel leader cannot discriminate among advantaged bidders through signal contingent side-payments, it directly follows that if the profit-maximizing cartel

[3] This is the standard assumption in contract theory. If relaxed, so that players indifferent between becoming a cartel member and staying out decide to stay out, then the stable cartel is still all-inclusive and led by the advantaged bidder, but the leader needs to leave strictly positive rents to his members.

المنارة للاستشارات
www.manaraa.com

leader sets advantaged players' share at zero, that is such that the participation constraint of the lowest possible signal binds, then an advantaged player finds it optimal to *not* join the cartel since he has a positive expected payoff by individually competing against the cartel in the auction (see subsection 1.4.1). Instead, the cartel leader may shut down a non-zero measure interval of signals so as to leave information rents to advantaged bidders, but then it is obviously not all inclusive. Consequently, if there is strictly more than one advantaged bidder, an all-inclusive cartel fails to be stable. Our next result summarizes.

Proposition 1.9. *Suppose that $|\mathcal{A}| \geq 2$. A stable cartel contains all regular bidders but fails to be all-inclusive.*

The unstability of the all-inclusive cartel with multiple advantaged bidders is good news for the seller since such cartel structure minimizes the seller's expected equilibrium revenue. In fact, absent any reserve price, his revenue goes down to zero. This suggests that, in an environment with strong asymmetries across bidders so that some bidders have a strictly higher ex-post valuation for the object for sale, the seller is better off when there are at least two advantaged bidders.

1.5 Competing cartels

Up to now, we focused on the formation of one cartel only so as to examine the composition of its membership in terms of types of bidders and its stability properties. We now allow for the formation of two cartels that will compete against each other in the auction. Suppose that we have two possibly asymmetric cartels, each led by

المنارة للاستشارات
www.manaraa.com

an advantaged bidder. Our next result characterizes the equilibrium at the auction stage.

Proposition 1.10. *Suppose that* $\pi = \{C_1, C_2\}$ *is efficient and* $|\mathcal{C}_1| = k_1$*,* $|\mathcal{C}_2| = k_2$ *such that* $n = k_1 + k_2$*. Then the following holds in equilibrium:*

(i) Each cartel bids $\beta(\bar{x}_i) = n\bar{x}_i$ *for all* $i \neq j$*,* $i, j = 1, 2$ *and neither suffers from ex-post regret,*

(ii) The seller's expected revenue is higher if $k_1 = k_2$*.*

Proof. The proof of (i) closely follows that of proposition 1.2. Consider cartel $\mathcal{C}_1$ (say) and suppose that cartel $\mathcal{C}_2$ bids according to the aforementioned strategy. By bidding $\beta(\bar{x}_1)$, $\mathcal{C}_1$ wins if $\bar{x}_1 \geq \bar{x}_2$ and gets $k_1\alpha(\bar{x}_1 - \bar{x}_2) \geq 0$, which ensures that it does not regret winning. Since the price is set by cartel 2, submitting a higher bid does not affect his payoff either so that playing $\beta(\bar{x}_1)$ is indeed optimal for cartel 1. In turn, cartel 2 does not regret losing as outbidding cartel 1 would yield $k_2\alpha(\bar{x}_2 - \bar{x}_1) \leq 0$. A similar argument shows that neither cartel suffers from ex-post regret whenever $\bar{x}_1 \leq \bar{x}_2$. The proof of (ii) is provided in Mares (2001). □

We now modify the previous notion of stability to account for possibly multiple cartels by adding a third stability criterion, the *inter-cartel stability* , which ensures that no cartel member has an incentive to switch to another cartel. We denote by $U(k_l + 1, k_i - 1)$ the ex-ante expected payoff of a member of cartel $\mathcal{C}_i$ when switching to cartel $\mathcal{C}_l$ (and therefore leaving $\mathcal{C}_i$), with $U(k_i + 1, k_i - 1) \equiv F^{c_i}(k_i, x)$, i.e. the payoff of a member of $\mathcal{C}_i$ that does not switch.

المنارة للاستشارات

www.manaraa.com

Definition 1.2. A cartel structure π is stable if each $\mathcal{C}_i \in \pi$ satisfies internal and external stability, and if for all $\mathcal{C}_i \in \pi$, the following holds:

Inter-cartel stability: $U(k_i+1, k_i-1) \geq U(k_l+1, k_i-1)$ for all $i \neq l$, $i, l = 1, 2$

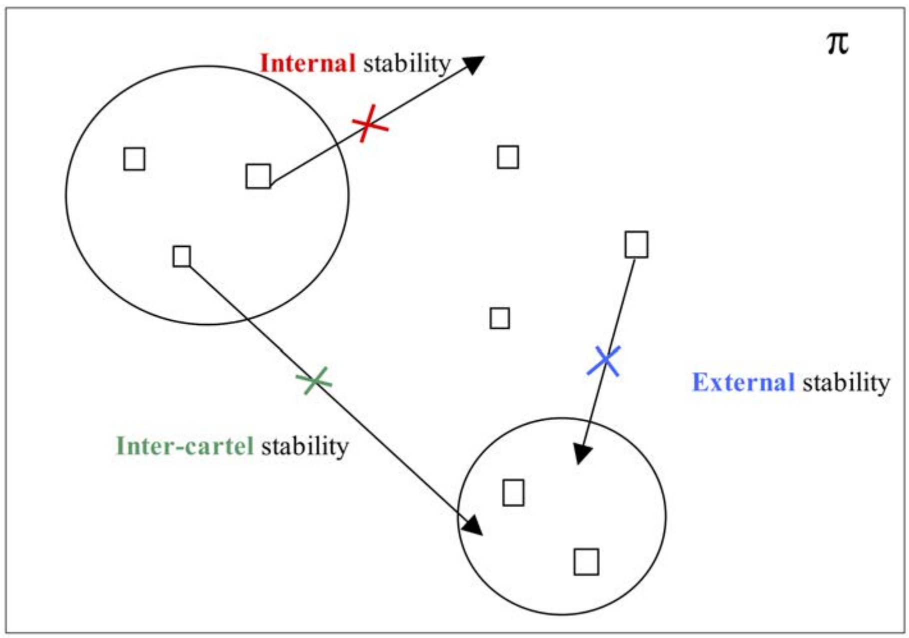

Figure 1.1: Stable cartel structure

Proposition 1.11. *Suppose that both cartels use the equal-sharing rule. If cartels have the same size, i.e. if $k_i = k_j$ for all $i, j \in \pi$, $i \neq j$, then the cartel structure is stable.*

Proof. Suppose that both cartels are of same size s. Given the symmetry of the problem, each cartel's ex-ante probability to win the auction is then $1/2$. In partic-

المنارة للاستشارات
www.manaraa.com

ular, they all have the same expected payoff since aggregate signals are identically distributed according to $F^{(*s)}$. Using the equal sharing rules, every advantaged member prefers to stay in its own cartel since switching to another one will decrease its side-payment. □

1.6 Conclusion

The practice of joint bidding, whereby a group of bidders submits a common bid, is a common phenomenon that has brought much concern regarding their potential negative anti-competitive effects. Merged bidders can pool their information and/or resources together. However, such practices also raise competition policy concerns since they reduce the degree of competition and may therefore harm the seller, but also as they may foster collusive price-setting behavior.

This paper examines the formation of bidding consortia in auctions in *almost* common-value auctions by endogenizing the cartel formation game. Cartel leaders discriminate between advantaged and regular players through their schedule of sharing rules since they greatly differ in their ability to win the auction on their own. If there is only one cartel, the cartel leader can attract regular bidders without leaving rents so that he acquires information about the common value at no cost.

With more than one advantaged bidder, an all-inclusive cartel is not stable since advantaged bidders have an incentive to stay out due to their positive probability to win the auction. Nevertheless, we show than any stable cartel must include all regular players. In the case of multiple competing cartels, a equal-sharing rule yields

المنارة للاستشارات
www.manaraa.com

a stable cartel structure if cartels are of same size.

www.manaraa.com

CHAPTER 2

COURNOT OLIGOPOLY AND AGGREGATE-POTENTIALS

(Joint with Rabah Amir)

2.1 Introduction

The present paper makes a methodological contribution to the basic theory of Cournot oligopoly by framing the central issues of existence, uniqueness and comparative statics in symmetric Cournot oligopoly through the maximization of an appropriate weighted sum of industry profits and social welfare. While the equivalence between the solutions of these two problems was first noted by Bergstrom and Varian (1985), we go beyond their treatment in allowing a much more general class of demand functions, which makes the correspondence between the argmax's of this weighted sum and the Cournot equilibria more subtle than the previously known one-to-one relationship (see Proposition 2.1). Since this forms an important part of the present paper, we elaborate in some detail on its contents.

We derive general answers to the above basic issues (i.e., existence, uniqueness and comparative statics) in Cournot oligopoly from the same framework in a novel manner. In particular, the problem of existence of symmetric Cournot equilibrium (usually treated as a fixed point of a best response mapping) is hereby converted into the much simpler problem of existence of an argmax for the weighted sum (via Weierstrass's Theorem). While the underlying weighted sum of industry profits and

المنارة للاستشارات
www.manaraa.com

social welfare is reminiscent of a potential for the Cournot game, it is actually quite distinct an entity since the weighted sum is maximized with respect to industry output as its sole argument[1]. On the other hand, the weighted sum used here may be regarded as a diagonal or aggregative potential for a symmetric Cournot game, and our novel use of this concept to settle the above basic issues for the Cournot model is also akin to some common usages of the notion of regular potential in strategic games (Monderer and Shapley, 1996).

In a similar vein, although usually framed as an issue of uniqueness of the fixed point of a mapping, the issue of uniqueness of Cournot equilibrium is converted to a problem of strict concavity of this aggregate potential. If the latter admits a unique argmax, then the associated Cournot game must have a unique Cournot equilibrium. This simple approach yields minimally sufficient conditions for a unique Cournot equilibrium that are already known in the literature, but reached through a novel and more elemantary argument.

As to the comparative statics property of Cournot equilibrium, they can be established in a fairly standard manner using some basic properties of the underlying potential maximization problem, instead of relying on the methodology dealing with the comparative statics of Nash equilibria (as in Amir and Lambson, 2000). We hereby recover the well-known results that for symmetric Cournot oligopoly with linear costs

[1]Recall that a potential for a strategic game is instead defined on the cartesian product of players' action spaces and has the property that any argmax constitutes a Nash equilibrium of the game (Monderer and Shapley, 1995). In contrast, here, an argmax of the weighted sum corresponds to the industry (total) output of some Cournot equilibrium.

المنارة للاستشارات

www.manaraa.com

and any inverse demand function, increased competition leads to a higher industry output, lower industry profits, and higher social welfare.

Last but not least, this perspective on the basic theory of Cournot oligopoly also allows one to derive general results on the second order properties of the equilibrium per-firm profit and social welfare functions, as they depend on their two key arguments here, namely the number of firms in the industry and on the (common) unit cost. Although such properties have often been used in industrial organization as basic assumptions of a reduced-form type to address various issues at a high level of generality, there are very few results in the extant literature that actually justify such assumptions. The reason behind this absence of results is that there are no general conclusions about the second order properties of equilibrium payoff functions in the parameters of a game. In this regard, this aspect of the paper may thus be seen as the most innovative part of the paper.

The paper is organized as follows. Section 2.2 describes the Cournot model and the novel approach to existence and uniqueness issues via the maximization of a fictitious objective function's. Section 2.3 provides the comparative statics results as firms' unit costs increase due to environmental regulation. Section 2.4 deals with the same questions for the case of an $n-$firm cartel. Section 2.5 provides a brief conclusion.

المنارة للاستشارات

www.manaraa.com

2.2 A maximization approach to Cournot equilibrium

Consider a Cournot industry with n identical firms producing a homogeneous good at a constant marginal cost $c > 0$ (common to all firms) and facing an inverse demand function $P(\cdot)$. Let q_i denote the output of firm i, $Q_{-i} = \sum_{j \neq i}^{n} q_j$ stand for the cumulative output of firm i's rivals, and Q the cumulative industry output, i.e., $Q = \sum_{i=1}^{n} q_i = q_i + Q_{-i}$.

The profit function of firm i is

$$\pi_i \left(q_i, Q_{-i}\right) = q_i \left[P\left(q_i + Q_{-i}\right) - c\right].$$

As usual, firm i's reaction correspondence is defined by

$$r(Q_{-i}) \triangleq \arg\max \left\{\pi_i \left(q, Q_{-i}\right) : q \geq 0\right\}$$

where the subscript i is dropped in view of symmetry.

An $n-$ tuple $(q^*, q^*, ..., q^*)$ is a symmetric Cournot equilibrium of the $n-$firm oligopoly if

$$q^* P(nq^*) - cq^* \geq qP[q + (n-1)q^*] - cq, \text{ for all } q \geq 0$$

The Marshallian measure of social welfare when industry output is Q is given as usual by

$$W(Q) \triangleq \int_0^Q P(t)dt - cQ$$

At equilibrium, the various variables at hand will be denoted by a star. Thus, for instance, equilibrium per-firm profit and social welfare will be denoted respectively by

المنارة للاستشارات
www.manaraa.com

π^* and W^*. We explicitly deal with the (possible) nonuniqueness of Cournot equilibria whenever the minimal assumptions we impose do not ensure uniqueness.

The following assumption is in effect throughout the paper:

(A1) *The inverse demand function* $P(\cdot)$ *satisfies:*

(i) $P(\cdot)$ *is twice continuously differentiable.*

(ii) $P'(\cdot) < 0$.

(iii) there exists $\overline{Q} > 0$ *such that* $P(\overline{Q}) = c$.

These conditions are minimal and their content is self-evident. Two immediate consequences need to be highlighted as they will be used repeatedly in the sequel.

The first is that **(A1)** (ii)-(iii) imply that $P(0) > c$, which in this context will guarantee that symmetric Cournot equilibria will be interior (i.e., with a strictly positive output level).

The second consequence is that, due to (A1), output levels higher than $\overline{Q} = P^{-1}(c)$ will not be observed under any market structure (in a Cournot setting, such output levels reflect the use of dominated strategies, for they would lead to negative mark-ups, and thus profits, regardless of what the rivals produce). Henceforth, we consider the set $[0, P^{-1}(c)]$ as the effective output (or action) set for every firm in the Cournot game. Clearly, $r(y) = 0$ for all $y \geq P^{-1}(c) = \overline{Q}$.

2.2.1 Existence of Cournot equilibrium via maximization

As in Bergstrom and Varian (1985), we define the following function

$$B_n(Q) \triangleq (n-1)\int_0^Q P(t)dt + QP(Q) - ncQ$$

المنارة للاستشارات
www.manaraa.com

For some of the uses of this function below, it is more convenient to work with the version obtained by dividing by n

$$\widetilde{B}_n(Q) \triangleq \frac{n-1}{n}\int_0^Q P(t)dt + \frac{1}{n}\left[QP(Q) - ncQ\right] \tag{2.1}$$

To provide a revealing interpretation of this function of industry output, observe that it can be equivalently rewritten in the form

$$\widetilde{B}_n(Q) \triangleq \frac{n-1}{n}[\int_0^Q P(t)dt - cQ] + \frac{1}{n}QP(Q) - cQ \tag{2.2}$$

in which case, as observed by Bergstrom and Varian (1985), it becomes transparent that $B(Q)$ is a weighted average of social welfare and industry profit, with n denoting the number of identical firms and Q the total output. In particular, the extreme values $n = 1$ and $n = \infty$ correspond respectively to monopoly and perfect competition. In addition, as n increases, the weight on social welfare increases at the expense of the weight on profits.

By a direct comparison of the associated first order conditions, Bergstrom and Varian (1985) proved that if $P(\cdot)$ is strictly concave, the *unique* maximizer of $B(Q)$ coincides with the industry output at the *unique* Cournot equilibrium of the corresponding Cournot game. They conclude that $B(Q)$ is a *fictitious objective function* for a Cournot oligopoly, in the sense that standard Cournot behavior amounts to the firms (tacitly) jointly maximizing $B(Q)$. So just as monopoly maximizes industry profit and perfect competition maximizes social welfare, an n-firm Cournot oligopoly behaves exactly as if it purported to maximize $B(Q)$.

In what follows, we generalize this result by removing the restriction of concave

المنارة للاستشارات
www.manaraa.com

inverse demand, in which case the connection between the maximizers of $B(Q)$ and the Cournot equilibria becomes somewhat more subtle (see upcoming result). In addition, we then use this new result to provide a new approach to the issues of existence and uniqueness of Cournot equilibria. For an overview of the existing literature on this topic, see Novshek (1985), Amir (1996), Gaudet and Salant (1991), and Amir and Lambson (2000), or Vives (1999).

Define the maximal value of this weighted objective as $B_n^*(c) \triangleq \max_{Q \geq 0} B(Q)$, or simply B^*.

We are now ready for one of the central results of this paper, which is of independent interest and describes in detail the precise relationship between the maximizers of the function $B(Q)$ and the symmetric Cournot equilibria of the associated oligopoly, without assuming their uniqueness.

Proposition 2.1. *Under Assumption (A1),*

(a) The function $B(Q)$ achieves its maximum.

(b) Any market output $Q^* \in \arg\max B(Q)$ is the industry output corresponding to a symmetric Cournot equilibrium $(q^*, q^*, ..., q^*)$ of the $n-$firm oligopoly, with $q^* = Q^*/n$.

(c) A symmetric Cournot equilibrium exists.

(d) There exists no asymmetric Cournot equilibrium.

(e) Every symmetric Cournot equilibrium $(q^*, q^*, ..., q^*)$ of the $n-$firm oligopoly leads to an industry output $Q^* = nq^*$, which is a local argmax of $B(Q, c)$.

Proof. (a) In seeking for an argmax of $B(Q)$ with respect to Q, one can clearly

المنارة للاستشارات
www.manaraa.com

restrict attention to the interval $[0, \overline{Q}]$, where $\overline{Q} = P^{-1}(c)$, as implied by the remark following the statement of Assumption (A1). Since $B(\cdot)$ is a continuous function and the effective domain $[0, \overline{Q}]$ is a compact set, $B(Q)$ achieves its maximum by Weierstrass's Theorem.

(b) If Q^* is an argmax of $B(Q)$, a deviation from $Q^* = nq^*$ to any other output, say $Q = q + (n-1)q^*$, cannot increase the optimal value of B, so that

$$\begin{aligned}
B^* &\triangleq \frac{n-1}{n}\int_0^{Q^*} P(t)dt + \frac{1}{n}Q^*P(Q^*) - cQ^* \\
&\geq \frac{n-1}{n}\int_0^{Q} P(t)dt + \frac{1}{n}QP(Q) - cQ \\
&= \frac{n-1}{n}\int_0^{q+(n-1)q^*} P(t)dt + \frac{1}{n}[q+(n-1)q^*]P(q+(n-1)q^*) - c[q+(n-1)q^*]
\end{aligned}$$

Writing $Q^* = nq^*$, and collecting the integral terms as well as the cost terms, the above inequality becomes

$$q^*P(nq^*) - cq^* \geq \frac{n-1}{n}\int_{nq^*}^{q+(n-1)q^*} P(t)dt + \frac{1}{n}[q+(n-1)q^*]P(q+(n-1)q^*) - cq \quad (2.3)$$

We now consider two separate cases:

Case 1: $q^* \leq q$. Then (2.3) continues as

$$\begin{aligned}
q^*P(nq^*) - cq^* \geq\ & \tfrac{n-1}{n}(q-q^*)P[q+(n-1)q^*] \\
&+ \frac{1}{n}[q+(n-1)q^*]P(q+(n-1)q^*) - cq \\
&= qP(q+(n-1)q^*) - cq
\end{aligned}$$

where the inequality uses the fact that P is a decreasing function. The equality then follows simply from collecting terms.

المنارة للاستشارات
www.manaraa.com

Case 2: $q^* \geq q$. Then (2.3) continues as

$$q^*P(nq^*) - cq^* \geq \frac{n-1}{n}\left[-\int_{q+(n-1)q^*}^{nq^*} P(t)dt\right]$$

$$+\frac{1}{n}[q+(n-1)q^*]P(q+(n-1)q^*) - cq$$

$$\geq \frac{n-1}{n}(q-q^*)P[q+(n-1)q^*]$$

$$+\frac{1}{n}[q+(n-1)q^*]P(q+(n-1)q^*) - cq$$

$$= qP(q+(n-1)q^*) - cq$$

where the second inequality uses the fact that P is a decreasing function. The equality then simply follows from collecting terms. Clearly, we have arrived at the same inequality as for the case $q^* \leq q$.

Hence, the above argument clearly proves that $(q^*, q^*, ..., q^*)$, with $q^* = Q^*/n$, forms a symmetric Cournot equilibrium.

(c) This follows directly from parts (a) and (b).

(d) If an asymmetric Cournot equilibrium, with total output $\widehat{Q}$, has one firm's output, say $q_i = 0$, then the first-order condition for firm i will be that $P(\widehat{Q}) \leq c$, a contradiction to the first order condition of some other firm, say j, $P(\widehat{Q}) + q_j P'(\widehat{Q}) = c$, since the latter implies $P(\widehat{Q}) > c$. Hence, all equilibrium outputs are strictly positive.

Suppose there is an asymmetric Cournot equilibrium, with total output $\widehat{Q}$, and two individual outputs $q_i > 0$, $q_j > 0$, and $q_i \neq q_j$. Then combining the first-order conditions for firms i and j, we have

$$P(\widehat{Q}) + q_i P'(\widehat{Q}) = P(\widehat{Q}) + q_j P'(\widehat{Q}) = c,$$

المنارة للاستشارات
www.manaraa.com

which implies that $q_i = q_j$, a contradiction.

Hence, all Cournot equilibria must be symmetric.

(e) The first-order condition for any one firm at a Cournot equilibrium $(q^*, q^*, ..., q^*)$ is

$$P(nq^*) + q^* P'(nq^*) = c \tag{2.4}$$

The corresponding second-order condition is

$$P'(nq^*) + q^* P''(nq^*) \leq 0 \tag{2.5}$$

The first-order condition for a local maximum of $B(Q)$, call it $\widetilde{Q}$, is

$$\frac{\partial B(Q,c)}{\partial Q} = \frac{(n-1)}{n} P(Q) + \frac{1}{n}[P(Q) + QP'(Q)] - c = 0$$

or

$$P(\widetilde{Q}) + \frac{1}{n} \widetilde{Q} P'(\widetilde{Q}) = c \tag{2.6}$$

A corresponding sufficient second-order condition for a local maximum of B is

$$(n+1)P(\widetilde{Q}) + \widetilde{Q} P''(\widetilde{Q}) < 0 \tag{2.7}$$

Identifying nq^* with $\widetilde{Q}$, it is seen by inspection that the two first order conditions (2.6) and (2.4) are the same, and that (2.7) is implied by (2.5). To see the latter point, multiply across (2.5) by n and then add $P'(nq^*) < 0$. Hence, the Cournot equilibrium $(q^*, q^*, ..., q^*)$ is such that nq^* is a local maximizer of the objective B. $\square$

At the level of generality of this Proposition, in view of the possible multiplicity of the argmax's of B and of Cournot equilibria, the two-way equivalence between

المنارة للاستشارات

www.manaraa.com

maxima of B and Cournot equilibria (reported by Bergstrom and Varian, 1985 under the assumption $P'' < 0$) breaks down. Global maxima of B always correspond to Cournot equilibria of the game, but the latter can be guaranteed to constitute only local maxima of B. Interestingly though, asymmetric Cournot equilibria are still ruled out with this minimal structure.

2.2.2 Uniqueness of Cournot equilibrium via maximization

This subsection deals with the issue of uniqueness of Cournot equilibrium. In general, this property requires another assumption beyond (A1).

(A2) $P(\cdot)$ satisfies the global condition

$$(n+1)P'(Q) + QP''(Q) < 0 \text{ for all } Q \in [0, \overline{Q}] \tag{2.8}$$

This assumption has commonly appeared in the literature on Cournot oligopoly in this form or some close variant of it. It is typically referred to as a stability condition, starting with Seade (1980). Indeed, in case of multiple equilibria, any Cournot equilibrium that satisfies (2.8) is locally stable in the sense of convergence of best reply Cournot dynamics conditional on starting the process close enough to the equilibrium. However, in the present context, condition (2.8) is postulated to hold in a global sense, in which case it guarantees the convergence of Cournot dynamics from any initial condition. The reason for this is that, as we now show, condition (2.8) in fact guarantees the uniqueness of Cournot equilibrium in the present setting.

Proposition 2.2. *(i) Under Assumptions (A1)-(A2), for any $n \geq 2$, an $n-$firm Cournot oligopoly has a unique Cournot equilibrium, which is thus symmetric.*

المنارة للاستشارات
www.manaraa.com

(ii) Under Assumptions (A1), for any sufficiently large n, an $n-$firm Cournot oligopoly has a unique Cournot equilibrium, which is thus symmetric.

Proof. (i) We first show that Assumption (A2) implies that $B(\cdot)$ is strictly concave in Q. Differentiating $B(Q)$ twice yields

$$\frac{\partial^2 B_n(Q)}{\partial Q^2} = (n+1)P'(Q) + QP''(Q)$$

which is < 0 if and only if (A2) holds. Hence (A2) implies that, for each fixed n, B is a strictly concave function of Q, globally on $[0, \overline{Q}]$. As a consequence, $B_n(Q)$ has a unique argmax, which is also the unique local maximum. Since every symmetric Cournot equilibrium is a local maximum of B by Proposition 2.1(e), the Cournot oligopoly has a unique symmetric Cournot equilibrium. Since no asymmetric Cournot equilibrium exists (by Proposition 2.1(d)), the conclusion follows.

(ii) Since P is twice continuously differentiable, $P' < 0$ and the effective output space is $[0, \overline{Q}]$, condition (2.8) always holds (without assuming *A2*) for n sufficiently large. Hence, uniqueness of Cournot equilibrium for large n follows from Part (i). □

Proposition (2.2)(i) can also be adapted from existing general results in the literature on uniqueness of Cournot equilibrium (see e.g., Kolstad and Mathiesen, 1987 or Gaudet and Salant, 1991). On the other hand, the simple proof given here, which exploits the uniqueness of the argmax for the maximization of the objective function B, instead of some contraction property of the best response mapping, is new to the literature concerned with uniqueness of Cournot equilibrium. Furthermore, Part (ii) has not been noted previously in the latter literature, and comes up as an

المنارة للاستشارات
www.manaraa.com

important point in establishing convergence of symmetric Cournot equilibrium (in case of multiple equilibria) to perfect competition below.

2.3 Entry dynamics

In this section, the effects of exogenous entry of additional firms into the market on Cournot equilibrium characteristics are analysed via the approach of the fictitious objective function. In short, the main questions are, what are the effects of exogenously increasing the number of firms on industry output (or price), per-firm and industry profits and social welfare?

Since the number of firms in the market will be treated as a variable parameter in this section, we rewrite $\widetilde{B}(Q)$ with explicit dependence on n

$$\widetilde{B}_n(Q) \triangleq \frac{n-1}{n} \int_0^Q P(t)dt + \frac{1}{n} QP(Q) - cQ$$

Proposition 2.3. *As n increases, for the maximal and minimal Cournot equilibria, the following properties hold for all n:*

(i) Industry output increases, i.e, $\overline{Q}_{n+1} \geq \overline{Q}_n$, and $\underline{Q}_{n+1} \geq \underline{Q}_n$.

(ii) Social welfare increases, i.e, $\overline{W}_{n+1} \geq \overline{W}_n$, and $\underline{W}_{n+1} \geq \underline{W}_n$.

(iii) Industry profit decreases, i.e., $(n+1)\pi_{n+1} \leq n\pi_n$.

(iv) Per-firm profit decreases, i.e., $\pi_{n+1} \leq \pi_n$.

Proof. (i) We will apply Milgrom-Shannon's (1994) version of Topkis's Theorem to the parametric optimization problem $\max_{Q \geq 0} B_n(Q)$. We must show that $B_n(Q)$ has the strict single-crossing property in $(Q; n)$, for which a sufficient condition is that $\widetilde{B}$

المنارة للاستشارات
www.manaraa.com

has strict increasing differences, i.e. that for any $Q' > Q$ and n, we have

$$\widetilde{B}_{n+1}(Q') - \widetilde{B}_{n+1}(Q) > \widetilde{B}_n(Q') - \widetilde{B}_n(Q),$$

or in other words,

$$\begin{aligned} & \frac{n}{n+1}\int_0^{Q'} P(t)dt + \frac{1}{n+1}Q'P(Q') - cQ' - \frac{n}{n+1}\int_0^{Q} P(t)dt - \frac{1}{n+1}QP(Q) + cQ \\ > & \frac{n-1}{n}\int_0^{Q'} P(t)dt + \frac{1}{n}Q'P(Q') - cQ' - \frac{n-1}{n}\int_0^{Q} P(t)dt - \frac{1}{n}QP(Q) + cQ \end{aligned}$$

Upon collecting terms,

$$\frac{n}{n+1}\int_Q^{Q'} P(t)dt + \frac{1}{n+1}[Q'P(Q') - QP(Q)] > \frac{n-1}{n}\int_Q^{Q'} P(t)dt + \frac{1}{n}[Q'P(Q') - QP(Q)]$$

or equivalently

$$\int_Q^{Q'} P(t)dt > [Q'P(Q') - QP(Q)] \tag{2.9}$$

To show that (2.9) holds, note that since P is strictly decreasing, we always have

$$\begin{aligned} \int_Q^{Q'} P(t)dt &> (Q' - Q)P(Q) \\ &> [Q'P(Q') - QP(Q)] \text{ since } Q' > Q \end{aligned}$$

which proves that (2.9) holds.

Then, by Milgrom-Shannon's Theorem, all the selections of the argmax of $B_n(Q)$, i.e., $\overline{Q}_n$ and $\underline{Q}_n$ are increasing functions of n, which concludes this part of the proof.

(ii) We consider the case $\overline{W}_{n+1} \geq \overline{W}_n$, and leave the fully analogous other case to the reader.

www.manaraa.com

Since $\overline{Q}_{n+1} \in \arg\max B_{n+1}(Q)$, we have $B_{n+1}(\overline{Q}_{n+1}) \geq B_{n+1}(\overline{Q}_n)$, or

$$n \int_0^{\overline{Q}_{n+1}} P(t)dt + \overline{Q}_{n+1} P(\overline{Q}_{n+1}) - (n+1)c\overline{Q}_{n+1} \geq n \int_0^{\overline{Q}_n} P(t)dt + \overline{Q}_n P(\overline{Q}_n) - (n+1)c\overline{Q}_n \tag{2.10}$$

Likewise, since $\overline{Q}_n \in \arg\max B_n(Q)$, we have $B_n(\overline{Q}_n) \geq B_n(\overline{Q}_{n+1})$, or

$$(n-1) \int_0^{\overline{Q}_n} P(t)dt + \overline{Q}_n P(\overline{Q}_n) - nc\overline{Q}_n \geq (n-1) \int_0^{\overline{Q}_{n+1}} P(t)dt + \overline{Q}_{n+1} P(\overline{Q}_{n+1}) - nc\overline{Q}_{n+1} \tag{2.11}$$

Adding up (2.10) and (2.11) and cancelling terms yields

$$\int_0^{\overline{Q}_{n+1}} P(t)dt - c\overline{Q}_{n+1} \geq \int_0^{\overline{Q}_n} P(t)dt - c\overline{Q}_n$$

or equivalently

$$\overline{W}_{n+1} \geq \overline{W}_n.$$

(iii) We consider the case $\overline{\pi}_{n+1} \geq \overline{\pi}_n$, and leave the fully analogous other case to the reader.

Since $\overline{Q}_{n+1} \in \arg\max_Q B_{n+1}(Q)$, we have $(n-1)B_{n+1}(\overline{Q}_{n+1}) \geq (n-1)B_{n+1}(\overline{Q}_n)$, or

$$\begin{aligned} & (n-1)n \int_0^{\overline{Q}_{n+1}} P(t)dt + (n-1)\overline{Q}_{n+1} P(\overline{Q}_{n+1}) - (n^2-1)c\overline{Q}_{n+1} && (2.12) \\ \geq \;& (n-1)n \int_0^{\overline{Q}_n} P(t)dt + (n-1)\overline{Q}_n P(\overline{Q}_n) - (n^2-1)c\overline{Q}_n && (2.13) \end{aligned}$$

Likewise, since $\overline{Q}_n \in \arg\max_Q B_n(Q)$, we have $nB_n(\overline{Q}_n) \geq nB_n(\overline{Q}_{n+1})$, or

$$\begin{aligned} & n(n-1) \int_0^{\overline{Q}_n} P(t)dt + n\overline{Q}_n P(\overline{Q}_n) - n^2 c\overline{Q}_n && (2.14) \\ \geq \;& n(n-1) \int_0^{\overline{Q}_{n+1}} P(t)dt + n\overline{Q}_{n+1} P(\overline{Q}_{n+1}) - n^2 c\overline{Q}_{n+1} && (2.15) \end{aligned}$$

المنارة للاستشارات
www.manaraa.com

Adding up (2.12) and (2.14) and cancelling terms yields

$$\overline{Q}_n P(\overline{Q}_n) - c\overline{Q}_n \geq \overline{Q}_{n+1} P(\overline{Q}_{n+1}) - c\overline{Q}_{n+1}$$

or equivalently

$$(n+1)\overline{\pi}_{n+1} \geq n\overline{\pi}_n.$$

(iv) This is a direct corollary of part (iii).

This completes the proof of Proposition (2.3). □

2.4 Convergence to perfect competition

It has long been observed that partial equilibrium models of imperfect competition tend to converge to their perfect competition counterparts as the number of firms competing in the same market (with a fixed demand curve) is increased indefinitely. For Cournot oligopoly, Ruffin (1971) pointed out that, to be valid, this Folk belief actually requires that the production process be characterized by (weakly) decreasing returns to scale, or that the firms' (common) average cost curve be weakly increasing. Otherwise, as the number of firms approaches infinity (while the demand curve remains fixed), each firm would be producing close to zero output, thus away from the minimal average cost point. This feature alone then rules out a perfectly competitive market as the limiting behavior. Ruffin's (1971) result was established under the customary set of overly restrictive assumptions, including the strict concavity of each firm's profit function in own output. These assumptions imply in particular that uniqueness of Cournot equilibrium holds for every n.

المنارة للاستشارات

www.manaraa.com

Here we generalize Ruffin's result to our minimally restrictive set of assumptions on the demand side, so that convergence of Cournot equilibium obtains even when there exist multiple Cournot equilibria (for sufficiently small values of n). Since we continue to assume constant returns to scale in production in this subsection, Ruffin's assumption of non-increasing returns to scale is clearly satisfied.

Formally, define perfect competition as being the market structure corresponding to the first best outcome, obtained by maximizing social welfare, i.e.,

$$\max_{Q\geq 0} W(Q) = \int_0^Q P(t)dt - cQ.$$

Clearly, one obtains the usual first order condition for social efficiency

$$P(Q^*) = c \quad \text{or} \quad Q^* = P^{-1}(c) \tag{2.16}$$

Since the second order condition for the maximization of welfare with respect to Q, i.e, $W''(Q) = P'(Q) < 0$, clearly holds globally, this first order condition (2.16) is sufficient for social optimality. Hence, the perfectly competitive outcome is unique and characterized by (2.16).

In addition, denoting with stars all the optimal market variables referring to the perfectly competitive outcome, industry profit and social welfare clearly satisfy

$$\Pi^* = Q^*[P(Q^*) - c] = 0 \text{ , and } W^* = \int_0^{Q^*} P(t)dt - cQ^*$$

Proposition 2.4. *The $n-$firm maximal and minimal Cournot equilibria converge monotonically to perfect competition as $n \to \infty$, in the sense that*

(a) $B_n(Q)$ converges uniformly to $W(Q)$

المنارة للاستشارات
www.manaraa.com

(b) $\overline{Q}_n \uparrow Q^*$, $\underline{Q}_n \uparrow Q^*$.

(c) $\underline{P}_n^* \downarrow c$ *and* $\overline{P}_n^* \downarrow c$

(d) $\overline{W}_n \uparrow W^*$ *and* $\underline{W}_n \uparrow W^*$

(e) $\overline{\pi}_n \downarrow \Pi^* = 0$ *and* $\underline{\pi}_n \downarrow \Pi^* = 0$.

Proof. (a) To show that, as $n \to \infty$, $B_n(Q)$ converges uniformly to $W(Q)$, note that, from (2.2), it is easy to see that for any $\varepsilon > 0$, there is an N large enough (independent of $Q \in [0, \overline{Q}]$) such that $n \geq N$ implies that

$$\frac{1}{n}\left[QP(Q) - cQ\right] < \varepsilon$$

and

$$\left|\frac{n-1}{n}\left(\int_0^Q P(t)dt - cQ\right) - \left(\int_0^Q P(t)dt - cQ\right)\right| < \varepsilon$$

Hence, $|B_n(Q) - W(Q)| < \varepsilon$ for all $Q \in [0, \overline{Q}]$ whenever $n \geq N$, i.e., $B_n(Q)$ converges uniformly to $W(Q)$.

(b) Since $B_n(Q)$ converges uniformly to $W(Q)$, by a well-known result in approximation theory (see Kall, 1986), we conclude that any limit point of a sequence of argmax's of $B_n(Q)$ will converge to an argmax of $W(Q)$, i.e. Q^*, which is unique here. Since $\overline{Q}_n$ and $\underline{Q}_n$ are increasing sequences (by Proposition (2.3)), they are convergent sequences. Since for large enough n, there is a unique Cournot equilibrium (by Proposition 2.2), the sequences $\overline{Q}_n$ and $\underline{Q}_n$ must have the same limit, which, by the just-cited general result, must be equal to Q^*. This ends the proof of Part (i).

(c) This part follows directly from the previous proof and (2.16).

(d) The monotonic convergence of the sequences $\overline{W}_n$ and $\underline{W}_n$ follows from

المنارة للاستشارات
www.manaraa.com

Proposition (2.3), and the rest of the proof is then a direct consequence of the proof of part (a).

(e) Industry profit at the maximal Cournot equilibrium (say) is given by $\overline{Q}_n[P(\overline{Q}_n) - c]$, which clearly converges to $Q^*[P(Q^*) - c] = 0$ as $n \to \infty$, since P is a continuous function. Since industry (or total) profit, which is given by $n\pi_n$, converges to 0 as $n \to \infty$, per-firm profit $\overline{\pi}_n$ must a fortiori go to 0 too. □

Corollary 2.5. *Any Cournot equilibrium selection of the $n-$firm oligopoly converges to perfect competition as $n \to \infty$, in the sense that*

(i) $Q_n \longrightarrow Q^*$ *and* $P_n^* \longrightarrow c$

(ii) $W_n \longrightarrow W^*$

(iii) $\pi_n \longrightarrow \Pi^* = 0.$

Furthermore, convergence is eventually monotonic (i.e., for all but a finite number of values of n) for each of the above sequences.

Proof. All three parts (i)-(iii) of the Corollary follow from the fact that there is a unique Cournot equilibrium for all values of n beyond a finite threshold value (see Proposition (2.2)). Indeed, for such values of n, monotonic convergence to perfect competition follows directly from Proposition (2.4). □

2.5 Second-order properties

This section derives some novel results on the inter-related second-order properties of the equilibrium social welfare and industry profit functions, to be denoted

www.manaraa.com

respectively by $W_n(c)$ and $\Pi_n(c)$.[2] Since there are no general results extending the classical envelope-type theorems to games, we proceed instead by exploiting the fact that $B_n^*(c)$ is the value of a maximization problem,

$$B_n^*(c) = \max_{Q \geq 0} \left\{ (n-1) \int_0^Q P(t)dt + QP(Q) - ncQ \right\}, \tag{2.17}$$

which is neverthless related to our two equilibrium functions under consideration by the relationship

$$B_n^*(c) = (n-1)W_n(c) + \Pi_n(c)$$

We will also simply write $B_n^* = (n-1)W_n + \Pi_n$ when the dependence on c is not stressed.

In terms of second-order properties, we have

Proposition 2.6. $B_n^*(c) = (n-1)W_n + \Pi_n$ *is*

(a) convex in n for each c ,

(b) convex in c for each n, and

(c) submodular in (n, c).

Proof. (a) Treating n as a real variable, the maximand in (2.17) is linear in n for each fixed Q. Hence, maximizing w.r.t Q amounts to taking the pointwise supremum of an infinite collection of linear functions. Hence, by Theorem 5.5 in Rockafellar (1970), $B_n^*(c)$ is convex in n.

[2]Since these two functions depend on two variables, c and n, the second order properties here will stand for the second variations w.r.t either variable as well as for the cross variation w.r.t. both variables.

المنارة للاستشارات

www.manaraa.com

(b) The maximand in (2.17) is linear in c for each fixed Q. Hence, maximizing w.r.t Q amounts to taking the pointwise supremum of an infinite collection of linear functions. Hence, By [Theorem 5.5 in Rockafellar, 1970), $B_n^*(c)$ is convex in c.

(c) We first recall that we can restrict attention to $Q \in [0, P^{-1}(c)]$ since all outputs $Q > P^{-1}(c)$ can be shown to lead to a strictly negative maximand, which means they are strictly dominated by the choice $Q = P^{-1}(c)$, which leads to a maximand of 0 (as is easy to check).

Next, we show that the maximand in (2.17) is supermodular in $(Q, -c, n) \in [0, P^{-1}(c)] \times (-\infty, 0] \times N$, or equivalently that the maximand in (2.17) has increasing differences w.r.t. every pair from the three variables. Using the smooth test, it can be easily checked that the cross partial of the objective function in (2.17) w.r.t $(Q, c), (c, n)$, and (Q, n) are respectively given by $-n, -Q$, and $P(Q) - c$, which are $< 0, \leq 0$ and ≥ 0 (the latter since $Q \in [0, P^{-1}(c)]$), respectively.

Hence, the maximand in (2.17) is supermodular in $(Q, -c, n)$. Since partial maximization preserves supermodularity w.r.t. the remaining variables, see [Tokpis, 1978, Theorem 4.3.], we conclude that $B_n^*(c)$ is supermodular in $(-c, n)$ or submodular in (c, n). □

Not surprisingly, there are no general, analogous results in games, the reason being that neither of the results used in the above proof, yielding convexity or supermodularity of a value function of a parametric optimization problem, here has any counterparts in games. This imparts particular interest to this result, which we now express through various special cases, as potentially useful (and more transparent)

المنارة للاستشارات
www.manaraa.com

implications of the proposition, directly on $W_n(c)$ and $\Pi_n(c)$.

Corollary 2.7. *$B_n^*(c) = (n-1)W_n + \Pi_n$ is convex in n, or for all $n \geq 1$,*

$$nW_{n+1} + \Pi_{n+1} + (n-2)W_{n-1} + \Pi_{n-1} \geq 2\left[(n-1)W_n + \Pi_n\right] \tag{2.18}$$

Proof. Writing the convexity w.r.t. n while treating n as a discrete variable, yields

$$[nW_{n+1} + \Pi_{n+1}] - [(n-1)W_n + \Pi_n] \geq [(n-1)W_n + \Pi_n] - [(n-2)W_{n-1} + \Pi_{n-1}]$$

which is the same as (2.18). □

Corollary 2.8. *$B_n^*(c) = (n-1)W_n(c) + \Pi_n(c)$ is convex in c for all $n \geq 1$. Or $(n-1)CS + n\Pi_n$ is convex in c and in n.*

In particular,

(i) the optimal monopoly profit, $\Pi_1(c)$, is convex in c.

(ii) for duopoly, $W_2(c) + \Pi_2(c)$ is convex in c.

(iii) for perfect competition, $W(c)$ convex in c, as convexity is preserved by limits

Proof. For (i), let $n = 1$, and for (ii), let $n = 2$. □

Corollary 2.9. *(i) For any $c' > c$, $[(n-1)W_n(c') + \Pi_n(c')] - [(n-1)W_n(c) + \Pi_n(c)]$ is decreasing in n.*

(ii) For any $n \geq 1$, $nW_{n+1}(c) + \Pi_{n+1}(c) - [(n-1)W_n(c) + \Pi_n(c)]$ is decreasing in c.

www.manaraa.com

2.6 Conclusion

The present paper deals with the basic theory of Cournot oligopoly, and aims at re-framing the central issues of existence, uniqueness and comparative statics in symmetric Cournot oligopoly through the maximization of an appropriate weighted sum of industry profits and social welfare.

Another objective of the paper is to elaborate on the notion that a symmetric Cournot oligopoly may be seen as maximizing a fictitious objective function of total industry output, as well as on its converse, and to derive basic general results for Cournot equilibrium in an elementary way from this relationship alone. In other words, we exploit the fact that symmetric Cournot equilibria (usually seen as fixed points of a suitable best-response mapping) may be converted into argmax's of a suitable objective function in order to derive basic results on existence, uniqueness and comparative statics in Cournot oligopoly.

Under this novel perpspective, we are able to rederive a number of existing results, as well as develop some general second-order properties for the equilibrium profit and social welfare functions with respect to the number of firms and the unit cost.

www.manaraa.com

CHAPTER 3

PRICE COMPETITION WITH DIFFERENTIATED GOODS AND INCOMPLETE PRODUCT AWARENESS

(Based on work joint with Filomena Garcia and Malgorzata Knauff)

3.1 Introduction

In retail economics, a market is said to be transparent if a large proportion of potential consumers are aware of the different products that are available, at what price and with which characteristics. Increasing transparency is often considered as a cure for some market imperfections and associated allocative inefficiencies that might otherwise arise. From the consumers' point of view, increased transparency is often believed to increase competition and thus consumer surplus, by generating lower prices and a reduction in price dispersion.

As a result of the emergence and the growing popularity of the Internet, which allows for instant access to relevant information in many markets, a global increase in market transparency is broadly believed to have taken place. Nevertheless, a large body of empirical research has provided mixed evidence on price comparisons between the Internet and traditional retailers. For instance, Bailey (1998) shows that Internet commerce may not reduce market friction because prices are higher when consumers buy homogeneous products on the Internet, and price dispersion for homogenous products among Internet retailers is greater than the price dispersion among physical

المنارة للاستشارات
www.manaraa.com

retailers. Lee et al. (2000) found that the average product price in one of the most successful electronic commerce systems (an electronic market system for used-car transactions in Japan) is much higher than in traditional, nonelectronic markets. The second-hand cars traded there are usually of much higher quality than those sold in traditional markets, but used-car prices are slightly higher than in traditional markets even for cars of similar quality. Conversely, Brynjolfsson and Smith (2000) observed that books and CDs in the internet are cheaper than in conventional outlets. They find that prices on the Internet are 9-16% lower than prices in conventional outlets and conclude that while there is lower friction in many dimensions of Internet competition, branding, awareness, and trust remain important sources of heterogeneity among Internet retailers.

In the theoretical literature there are studies explaining the phenomenon that prices do not always go down in case of increased transparency. The main argument, invoked in a number of recent papers, is that increasing transparency might facilitate tacit collusion for the producers (see e.g. Mollgaard and Overgaard, 2001, Nilsson, 1999, and Schultz, 2005).

The problem of market transparency was also investigated in many different related strands of literature. Varian (1980) showed that in case of homogenous goods and symmetric firms, the expected equilibrium profits decrease in the level of market transparency. This idea was developed in the search literature, for instance Burdett and Judd (1983) or Stahl (1989). If the cost of searching goes down, the consumers search more and inter-firm competition becomes tougher. Another approach to ex-

المنارة للاستشارات
www.manaraa.com

plaining market transparency issues can be found in the literature on advertising, with increased advertising typically leading to lower prices, as for instance in Bester and Petrakis (1995) where transparency is considered as a firm's decision variable. A special case of this literature is Ireland's (1993) model on information provision in price competition with homogenous goods, for which there are only asymmetric pure strategy Nash equilibria and the firm with higher information provision charges on average a larger price.

Another strand of literature studies the demand side of the market under less than full transparency. For the Hotelling model with product differentiation and a fraction of uninformed consumers, Schultz (2004) shows that increasing transparency (measured by the proportion of informed consumers) leads to less product differentiation and lower prices and profits. Moreover, welfare improves for all consumers and total surplus increases. Boone and Potters (2002) analyzed a symmetric Cournot-Nash model, where goods are imperfect substitutes and consumers value variety. They found that more transparency may lead to an increase in total demand and also to higher prices. The level of substitutability is exogenous in their model and, when goods are perfect substitutes, the effect of increasing demand disappears.

The model presented in this paper is closely related to this last strand of literature. We deal with effects of market transparency on prices in the standard Bertrand duopoly model with heterogeneous goods, modified to allow for transparency effects. In contrast to Bester and Petrakis (1995), transparency is viewed in the present paper as a characteristic of the industry under consideration, thus as an exogenous

www.manaraa.com

parameter. The analysis is intuitive and simple when we consider two types of strategic interaction between firms' prices in the industry - strategic complementarity and strategic substitutability.

We derive our results in the form of equilibrium comparative statics analysis, using the methodology of supermodular games (see Vives, 1999 and Amir, 2005 for general surveys of this methodology as applied to oligopoly theory).[1] This framework is very natural for the issues under consideration in the present paper. It allows for a resolution of the main questions of interest under minimal sufficient conditions. This parsimony in the required assumptions allows for easy and insightful interpretations of our findings.

In the first case, with prices being strategic complements, the results conform closely with conventional wisdom, especially, if in addition products are assumed to be gross substitutes. Namely, equilibrium prices are always decreasing in the transparency level. This is the intuitive conclusion, one that is often advanced in policy circles as reflecting the natural effects of the Internet and other advances in information technology.

Considering price competition with strategic substitutes, an ambiguity in the direction of change of prices appears. This is due to the fact that the Bertrand game is then a game of strategic substitutes, for which it is well known that downward shifts

[1]The general theory was pioneeered by Topkis (1978), and surveyed in full detail in Topkis (1998). Other specific applications of lattice programming to oligopoly theory include Vives (1990), Amir (1996a), Amir and Lambson (2000), Hoernig (2003), and Prokopovych and Yannelis (2017), among others.

المنارة للاستشارات

www.manaraa.com

in reaction curves need not always translate into lower equilibrium prices. Therefore, the lack of a definite result is easily predictable in light of the general results in the theory of supermodular games. Nevertheless from a purely intuitive standpoint, this is less of a natural conclusion. This indetermination in price changes subsequently leads to ambiguity concerning equilibrium profits and surplus changes a result of increasing transparency as well.

When the demand function is specialized to the standard linear demand for differentiated products, a complete characterization of the properties of the duopoly with uninformed consumers becomes possible. In particular, the equilibrium statics properties of market performance with respect to changes in the transparency level are fully derived, including the effects on equilibrium prices, profits and social welfare.

The remainder of this paper is organized as follows. In Section 3.2 we present the general set-up of the model of price competition with incomplete transparency. In Section 3.3 we study the reaction of Bertrand equilibrium prices to increased market transparency, distinguishing the two cases of strategic complementarity and substitutability. In Section 3.4 we complement the results of other sections for the special case of linear demand for differentiated products, which enables a full characterization. A brief conclusion follows.

3.2 Set-up and definitions

In this section, we lay out the general model of price competition modified in a way that integrates the transparency issue in a natural way. Our model is a

المنارة للاستشارات
www.manaraa.com

generalization of Schultz (2004) and Boone and Potters (2006). We also provide a microeconomic foundation for the unusual demand system under consideration in the usual representative consumer framework.

3.2.1 The model

We consider a Bertrand price competition game Γ with the following characteristics. Two firms, producing differentiated products, respectively 1 and 2, compete in prices. Firms 1 and 2 have constant marginal costs c_1 and c_2 respectively.

Following Schultz (2004) and Boone and Potters (2006), we consider two different types of consumers. A fraction ϕ of the consumers are informed about both products (both in terms of characteristics and prices) and the rest, the fraction $1-\phi$, are completely uninformed about one of the products. The parameter $\phi \in [0,1]$ is thus a natural measure of the level of transparency of the market.

Schultz considered the Hotelling model with a continuum of consumers uniformly distributed along the interval $[0,1]$ and the demand for firm 1's product is given by $\phi x + (1-\phi)\frac{1}{2}$, where $x \in [0,1]$ denotes the location of the consumer who is fully informed and indifferent between buying product 1 and 2. The demand for firm 2's product is $1 - \left(\phi x + (1-\phi)\frac{1}{2}\right)$.

We generalize this approach by allowing for other forms of demand functions, but retain the same way of modelling the behavior of informed versus uninformed consumers. We consider a one-shot model with exogenous heterogeneity of the products and firms deciding only on prices.

www.manaraa.com

The full-information demands for goods 1 and 2 are denoted respectively $D^1(p_1, p_2)$ and $D^2(p_2, p_1)$. The uninformed consumers know only about one of the products and are not even aware of the existence or the presence of the other product. Hence, these uninformed consumers' demands depend only on the price of the one good they know about, say good i, or $d_i(p_i)$, $i = 1, 2$. We assume that half of the uninformed consumers know about each of the two goods, so we posit that these consumers end up allocating themselves equally across the two firms. Thus, the total demand for good i is[2]

$$\phi D^i(p_i, p_j) + \frac{1-\phi}{2} d_i(p_i).$$

We assume throughout that $D^i(p_i, p_j)$ and $d_i(p_i)$ are twice continuously differentiable. The two goods are gross substitutes if the demand for either one of them is globally strictly increasing in the other's price, i.e. when $D^i_j > 0$.[3] The two goods are gross complements if the demand for either one of them is globally strictly decreasing in the other's price, i.e. when $D^i_j < 0$. Finally, the two goods are independent if the demand for either one of them is independent of the other good's price.

We shall also consider goods with a general relationship, i.e. demand systems where the two goods are neither substitutes nor complements in a global sense, i.e. goods for which D^i_j changes signs as the cross price p_j varies.

[2]The present model is a generalization of the linear model by Boone and Potters (2006) in the two-good case. In Schultz (2004), the demands $d_i(p_i)$ are perfectly inelastic since every uninformed consumer always buys one unit of the good, given a fully covered Hotelling market.

[3]For multivariate functions, subscripts denote partial derivative taken with respect to the indicated variable, here e.g. $D^i_j = \frac{\partial D^i}{\partial p_j}$.

المنارة للاستشارات
www.manaraa.com

Finally, the four demand functions $D^i(p_i, p_j)$ and $d_i(p_i)$ can be characterized by their price elasticities, respectively defined in the usual way by

$$\varepsilon_{D^i} = D^i_i \frac{p_i}{D^i} \text{ and } \varepsilon_{d_i} = d_i'(p_i)\frac{p_i}{d_i}, i = 1, 2.$$

Consider the situation where the level of market transparency is zero (i.e., $\phi = 0$). Then every firm would face half of the consumers, and there would be no relation between firms' pricing decisions, so firm i's profit would be given by

$$\widehat{\pi}^i(p_i) = \frac{1}{2}\left(p_i - c_i\right) d_i(p_i).$$

If we assume strict quasi-concavity of $\widehat{\pi}^i$ the unique solution of the profit maximization problem in this case, $\mathring{p}_i$, is given by the first order condition:

$$\widehat{\pi}^i_i(p_i) = d_i(\mathring{p}_i) - \left(\mathring{p}_i - c_i\right) d_i'(\mathring{p}_i) = 0$$

At the other extreme, when the market is perfectly transparent, i.e. all consumers are informed about prices and characteristics of both goods, the profit of firm i can be expressed in the standard way as:

$$\pi^i(p_i, p_j) = \left(p_i - c_i\right) D^i(p_i, p_j), i \neq j, i, j \in \{1, 2\}.$$

In case of imperfect market transparency, which is the situation of particular interest for the present paper, the profit of firm i is given by

$$\Pi^i(p_i, p_j) = \left(p_i - c_i\right)\left\{\phi D^i(p_i, p_j) + \frac{1-\phi}{2} d_i(p_i)\right\}. \tag{3.1}$$

This reflects the tacit assumption that firms are not allowed to price discriminate across the two types of consumers.

www.manaraa.com

We restrict our consideration to prices in $[c_i, \infty), i = 1, 2$, since lower prices are dominated by pricing at marginal cost. Moreover we assume an upper bound on price, $\overline{p}_i$, such that $p_i \in P_i = [c_i, \overline{p}_i]$.

The next subsection discusses possible theoretical foundations for the demand system at hand.

3.2.2 Microeconomic foundations for the demand system

It is customary in industrial organization to think of demand as being derived from a representative consumer maximizing a quasi-linear utility function of the two goods under consideration and a numeraire good, subject to a standard budget constraint. In the next subsection we shall explore two possibilities for such a foundation for the demand system at hand.

3.2.2.1 Option 1

The first option adapts the approach followed by Boone and Potters (2006) to our general demand set-up. For the informed sector, i.e. Type I consumers, consider a representative consumer with utility function $U(x_1, x_2) + y$, where x_i is the demand for good i and y is a composite commodity for all goods other than 1 and 2, whose price is normalized to 1.

We shall assume that U satisfies the following standard assumption:

(A1) (i) $U(x_1, x_2)$ is twice continuously differentiable,

(ii) $U(x_1, x_2)$ is differentiable strictly increasing, i.e., $U_1 > 0$ and $U_2 > 0$.

المنارة للاستشارات
www.manaraa.com

(ii) $U(x_1, x_2)$ is differentiable strictly concave, i.e.,

$$U_{11} < 0, U_{22} < 0, \text{ and } U_{11}U_{22} - U_{12}^2 > 0.$$

The representative consumer's problem is

$$\max\{U(x_1, x_2) + y : x_1, x_2, y\}$$

subject to

$$p_1x_1 + p_2x_2 + y \leq I,$$

where I is the exogenously given income level of the consumer.

This yields the inverse demand system (D^1, D^2) through the solution of the first-order conditions

$$\partial U(x_1, x_2)/\partial x_i = P^i(x_1, x_2), \quad i = 1, 2. \tag{3.2}$$

The direct demand system (D^1, D^2) is then obtained as the inverse of (P^1, P^2).

Then for those consumers in the uninformed sector who are aware only of good 1 (say), i.e. Type 1 consumers, being unaware of the presence of good 2, their problem may be stated as

$$\max U(x_1, 0) + y \tag{3.3}$$

subject to

$$p_1x_1 + y \leq I,$$

The resulting demand $d_1(p_1)$ is then derived as the inverse of $P_1(x_1, 0)$, the solution to (3.3), which satisfies the first-order condition

$$\partial U(x_1, 0)/\partial x_1 = P^1(x_1, 0). \tag{3.4}$$

www.manaraa.com

Likewise, Type 2 consumers solve $\max U(0, x_2) + y$ subject to the budget constraint $p_2 x_2 + y \leq I$, thereby giving rise to the other demand $d_2(p_2)$, via inversion of the solution to the first order condition

$$\partial U(0, x_2)/\partial x_2 = P^2(0, x_2).$$

Clearly, the two demand functions $d_2(p_2)$ and $d_1(p_1)$ are independent, but they are related via their origin from the same utility function U. In particular, the demands $d_1(\cdot)$ and $d_2(\cdot)$ will be identical if U is a symmetrical function of x_1 and x_2.

The advantage of this formulation is that all the demand functions at hand may be regarded as originating from the same representative consumer, depending only on his level of informativeness or awareness of the product space. In other words, the demands (D^1, D^2) and (d_1, d_2) may be seen as being consistent with each other. This is a meaningful property, which also makes a welfare analysis possible, as we shall see below.

A specific formulation along these lines appears in Boone and Potters (2006) with a quadratic utility function and thus linear demands (for differentiated products). The present treatment may be viewed as an extension of their formulation.

Yet, one may well argue that this construction is unnecessarily restrictive in that it fails to capture some economically meaningful situations to which the model at hand might apply, which takes us to the second option.

المنارة للاستشارات
www.manaraa.com

3.2.2.2 Option 2

Identify Type I consumers as those with access to the duopoly market, and Type i as those without access to the duopoly market, but with access only to a local market served by firm i only, $i = 1, 2$. A prime example would be Internet shoppers as Type I and traditional buyers who patronize only actual physical shops as Type i, for instance due to lack of access to the Internet or to credit cards. (These buyers' behavior can then either be justified on informational grounds as before, or on other grounds, such as prohibitive geographical distance from the shop offering the other product.) Other meaningful distinctions between the two classes of consumers might be urban vs rural consumers, market insiders vs outsiders, rich and poor, domestic vs international in a border market, etc... In light of the many possible sources of heterogeneity between these two classes of consumers, there is less compelling reason to presume that one representative consumer, i.e. one utility function, could approximate the behavior of the overall pool of buyers. One option then is to have one representative consumer for each of the three types of consumers: Type I, Type 1 and Type 2. Let the Type I consumer have the same optimization problem as before, thus yielding the demand system (D_1, D_2). Define type i's problem, for $i = 1, 2$, as $\max \widetilde{U}_i(x_i) + m$ subject to the budget constraint $p_i x_i + m \leq I$, for some valid utility function $\widetilde{U}_i(\cdot)$ and assume that this yields the demand $d_i(p_i)$, which is then unrelated to the demand D_i of the informed sector.

Nonetheless, one may still bring this setting within the spirit of a representative consumer, but in a framework of uncertainty. The underlying random representative

المنارة للاستشارات
www.manaraa.com

consumer shall have utility U with probability ϕ, and utilities $\widetilde{U}_1$ and $\widetilde{U}_2$ with probability $(1-\phi)/2$ each. It is easily verified that the objective functions given in (3.1) represent the firms' expected profit functions when they face this randomly drawn representative consumer.

This second formulation is clearly more general than the first, and allows for a wider scope of economic situations that fit the model. At the same time, being so broad, it is also consistent with uninteresting situations, such as when the uninformed sector is simply too small to matter.

The approach tacitly followed in this paper will be to treat the four demand functions as basic primitives, so that both of the above formulations can be accommodated, but specific references shall be made below mostly to the first approach since the consistency of the resulting demand functions allows for a welfare analysis and some comparisons of interest.

3.3 Effect of transparency on prices

In this section we consider the impact of increasing market transparency on equilibrium prices in the model formulated in (3.1). We distinguish two natural cases of analysis, depending on the character of the strategic interactions between firms.

3.3.1 Prices as strategic complements

Consider a Bertrand game with perfect transparency and payoffs given by

$$\pi^i(p_i, p_j) = (p_i - c_i)D^i(p_i, p_j).$$

www.manaraa.com

A sufficient condition on the market primitives that makes Bertrand competition a strictly supermodular game follows from setting the cross-partial derivative of the profit function (with respect to the two prices) non-negative. This condition is (see Vives, 1990):

$$D^i_j + (p_i - c_i)D^i_{ij} > 0 \text{ for all } (p_i, p_j) \in P_i \times P_j. \tag{3.5}$$

This condition is more easily satisfied when the products are substitutes (i.e. $D^i_j > 0$) and when demand is supermodular (i.e. $D^i_{ij} > 0$), but inspection of the terms involved easily reveals that neither of these is a necessary condition (although at least one of them must hold for each price pair).[4] For the widely used case of linear demands, it is well-known that prices are strategic complements (substitutes) in the standard Bertrand game if and only if products are substitutes (complements); see e.g., Singh and Vives (1984). However, with general demand functions, strategic complementarity of the game may hold even when the goods are complements. In the latter case, it is necessary that demand be strongly supermodular in prices, i.e. that D^i_{ij} be strongly > 0 (more on this point below). Furthermore, while most studies in industrial organization posit that goods are either substitutes or complements, we know from standard microeconomic theory that this need not be the case, in other words that goods need not have any such relationship in a global sense (i.e., for all feasible price pairs). As a consequence, when working with general demand functions,

[4]There is another condition making Bertrand duopoly with linear cost into a game of strategic complementarities. It was given by Milgrom and Roberts (1990) and is equivalent to the cross partial derivative of the log-profit function being non-negative. In our case it is less useful, since it requires imposing additional conditions on the game with imperfect transparency to secure its log-supermodularity.

www.manaraa.com

it is important to separate the relationship between the two goods in demand from the strategic complements/substitutes property of the resulting Bertrand game. The present paper shall adopt this line of thinking.

We now note that condition (3.5) guarantees the supermodularity of the game with imperfect transparency as well. This is clearly due to the separability of the overall profit function and the fact that the profit from the uninformed consumers depends only on one of the prices. Define the firms' price reaction correspondences as usual by

$$r_i(p_j) = \arg\max_{p_i} \left\{ (p_i - c_i) \left[\phi D^i(p_i, p_j) + \frac{1-\phi}{2} d_i(p_i) \right] \right\}$$

Lemma 3.1. Π^i *defined in (3.1) has strictly increasing differences in* $(p_i, p_j) \in P_i \times P_j$ *if* D^i *satisfies*

$$D^i_j + (p_i - c_i) D^i_{ij} > 0. \tag{3.6}$$

Hence, every selection of $r_i(p_j)$ *is strictly increasing in* p_j*, when interior.*

Proof. The cross-partial derivative of overall the profit function $\Pi^i(p_i, p_j)$ is given by

$$\Pi^i_{ij}(p_i, p_j) = \phi \left(D^i_j + (p_i - c_i) D^i_{ij} \right)$$

Setting $\Pi^i_{ij} > 0$ yields the result.

Since $\Pi^i_{ij} > 0$ implies that $\partial \Pi^i_i(p_i, p_j)/\partial p_i$ is strictly increasing in p_j, we conclude that every selection of $r_i(p_j)$ is strictly increasing in p_j, when interior, by a strengthening of Topkis's Theorem due to Amir (1996b) or Edlin and Shannon (1998). □

المنارة للاستشارات
www.manaraa.com

The supermodularity of the profit functions, or the strategic complementarity of the pricing game, is sufficient to guarantee the existence of a pure-strategy Bertrand equilibrium, even when profit functions are not quasi-concave in own price (i.e. when reaction curves are not necessarily continuous functions). Furthermore, it is also a key property in order to establish monotone comparative statics of equilibrium prices in reaction to a change in the level of market transparency.[5]

Before stating the main result on the price effects of increased transparency, we note that, at an intuitive level, there are two conflicting effects. The first is that for each firm, higher transparency reduces its monopoly power since it shifts consumers from the uninformed to the informed (duopoly) sector, thus putting downward pressure on its price. At the same time, the same effect also takes place for its competitor, thus resulting in more consumers in the duopoly sector that the firm might newly serve. The latter effect favors a higher price. In light of these conflicting effects, it is not surprising that the result is that either effect can dominate, depending on the relative demand elasticities in the two sectors. We state the result for the intuitive direction, but then discuss both possibilities below.

Proposition 3.2. *Assume that*

(i) $D^i_j + (p_i - c_i)D^i_{ij} > 0, i, j = 1, 2, i \neq j$ *and*

(ii) $|\varepsilon_{D^i}| \geq |\varepsilon_{d_i}|$ *for all* $(p_i, p_j) \in P_i \times P_j, i = 1, 2.$

[5] It is worthwhile to note here that other, more general complementarity conditions such as the single-crossing property (Milgrom and Shannon, 1994) or the interval dominance order (Quah and Strulovici, 2009) do not appear to be applicable in the present setting, due to the fact the profit function is a sum of different terms.

المنارة للاستشارات

www.manaraa.com

Then

(a) a Bertrand equilibrium exists for all values of ϕ, and

(b) an increase in market transparency ϕ causes the extremal equilibrium prices of both goods to decrease.

Proof. (a) From Lemma 3.1, we know that Π^i has strictly increasing differences in (p_i, p_j), so we have a strictly supermodular game for each value of the transparency parameter ϕ. Therefore, the existence of a pure-strategy Bertrand equilibrium follows directly from Tarski's fixed-point theorem.

(b) For the comparative statics result, it turns out to be more insightful to consider when $\Pi^i(p_i, p_j, \phi)$ is log-submodular, rather than simply submodular, in (p_i, ϕ). To this end, observe that

$$\log \Pi^i(p_i, p_j, \phi) = \log(p_i - c_i) + \log\left\{\phi D^i(p_i, p_j) + \frac{1-\phi}{2} d_i(p_i)\right\}.$$

To see when $\log \Pi^i(p_i, p_j, \phi)$ is submodular in (p_i, ϕ), we use Topkis's differential characterization. To do so, first consider

$$\frac{\partial \log \Pi^i(p_i, p_j, \phi)}{\partial \phi} = \frac{D^i(p_i, p_j) - \frac{1}{2} d_i(p_i)}{\phi D^i(p_i, p_j) + \frac{1-\phi}{2} d_i(p_i)}.$$

Differentiating next w.r.t. p_i, we have, upon simplification

$$\frac{\partial^2 \log \Pi^i(p_i, p_j, \phi)}{\partial p_i \partial \phi} = \frac{1}{2} \frac{D^i_i(p_i, p_j) d_i(p_i) - D^i(p_i, p_j) d_i'(p_i)}{\left[\phi D^i(p_i, p_j) + \frac{1-\phi}{2} d_i(p_i)\right]^2}. \tag{3.7}$$

Hence, $\partial^2 \log \Pi^i(p_i, p_j, \phi)/\partial p_i \partial \phi \leq 0$ whenever

$$D^i_i(p_i, p_j) d_i(p_i) - D^i(p_i, p_j) d_i'(p_i) \leq 0. \tag{3.8}$$

المنارة للاستشارات

www.manaraa.com

Dividing (3.8) by $D^i(p_i, p_j)d_i(p_i)$ and multiplying by p_i yields $\varepsilon_{D^i} - \varepsilon_{d_i} \leq 0$. Since both these elasticities are negative, the condition $\varepsilon_{D^i} - \varepsilon_{d_i} \leq 0$ is the same as $|\varepsilon_{D^i}| \geq |\varepsilon_{d_i}|$. We have just shown that the latter condition implies that $\log \Pi^i(p_i, p_j, \phi)$ is submodular in (p_i, ϕ).

We conclude via [Milgrom and Roberts, 1990, Theorem 5] that, when ϕ goes up, both prices go down for the maximal and the minimal equilibria of the pricing game at hand. $\square$

We now provide a discussion of the scope of the Proposition. Recall that the condition $D^i_j + (p_i - c_i)D^i_{ij} > 0$ is much easier to satisfy when the two goods at hand are substitutes since D^i_j is then > 0. All that is needed then is for the cross partial D^i_{ij} not to be too negative, a property satisfied by most commonly used demand functions (see Vives, 1999 for further discussion). It turns out that the elasticity condition $|\varepsilon_{D^i}| \geq |\varepsilon_{d_i}|$ is also more compatible with substitute products than with complementary products. In fact, as seen in the analysis of the special case of linear demand below, $|\varepsilon_{D^i}| \geq |\varepsilon_{d_i}|$ always holds for substitutes while the reverse condition holds for complements (under linear demand).

At a more intuitive level, the condition $|\varepsilon_{D^i}| > |\varepsilon_{d_i}|$ is quite natural for the model at hand, irrespective of which of the two representative consumer frameworks one takes. Indeed, the condition simply says that the demand for good i is more sensitive to changes in price for those consumers who are aware of the presence of both goods in the market. These consumers have the option of reacting to the price increase by switching to the other good, whereas those that are uninformed are not

المنارة للاستشارات
www.manaraa.com

aware of this possibility.

In Schultz's (2004) Hotelling model with full market coverage, the uninformed buyers are posited to always buy exactly one unit of the good, so that their demand is perfectly inelastic and the elasticity condition, $|\varepsilon_{D^i}| \geq |\varepsilon_{d_i}|$ for all (p_i, p_j), then trivially holds. It is easily verified that the latter elasticity condition also holds for the standard linear demand for substitute products of Boone and Potters (2006).[6]

3.3.2 Prices as strategic substitutes

This subsection explores the extent to which a similar result is possible when the price game at hand displays strategic substitutes. It is well known that in such a case, there are no general comparative statics result for Nash equilibria for asymmetric games (see Milgrom and Roberts, 1990 or Amir, 2005).

Analogously to Lemma 3.1, we can formulate a condition on D^i to make the price game Γ a submodular game.

Lemma 3.3. Π^i *defined like in (3.1) has strictly decreasing differences in* $(p_i, p_j) \in P_i \times P_j$ *if* D^i *satisfies*

$$D^i_j + (p_i - c_i)D^i_{ij} < 0. \tag{3.9}$$

Hence, every selection of $r_i(p_j)$ *is strictly decreasing in* p_j*, when interior.*

Proof. The cross-partial derivative of overall the profit function $\Pi^i(p_i, p_j)$ is given by

$$\Pi^i_{ij}(p_i, p_j) = \phi \left(D^i_j + (p_i - c_i)D^i_{ij} \right)$$

[6] In their model with Cournot competition, the fact that equilibrium prices may increase in the level of transparency is due entirely to their assumption of strongly decreasing returns to scale in production (or quadratic cost function).

المنارة للاستشارات

www.manaraa.com

Setting $\Pi^i_{ij} < 0$ yields the result.

The second statement follows as in the proof of Lemma 3.1 $\square$

The effect of a change in transparency on prices is captured in the next result.

Proposition 3.4. *Assume that*

(i) $D^i_j + (p_i - c_i)D^i_{ij} < 0, i, j = 1, 2, i \neq j$ *and*

(ii) $|\varepsilon_{D^i}| \leq |\varepsilon_{d_i}|$ *for all* $(p_i, p_j) \in P_i \times P_j, i = 1, 2.$

Then

(a) a Bertrand equilibrium exists for all values of ϕ*, and*

(b) an increase in market transparency ϕ *causes the extremal equilibrium price of at least one good to increase.*

Proof. (a) From the previous Lemma, we know that Π^i has strictly decreasing differences in (p_i, p_j), so we have a strictly submodular game for each value of the transparency parameter ϕ. Therefore, the existence of a pure-strategy Bertrand equilibrium follows directly from Tarski's fixed-point theorem applied to the composition of the two reaction correspondences (see Vives, 1990).

(b) For the comparative statics result, it is easy to see from the proof of Proposition 3.2 that $\partial^2 \log \Pi^i(p_i, p_j, \phi)/\partial p_i \partial \phi \geq 0$ if and only if $D^i_i(p_i, p_j)d_i(p_i) - D^i(p_i, p_j)d'_i(p_i) \geq 0$, and that the latter condition is the same as our elasticity assumption here, namely $|\varepsilon_{D^i}| \leq |\varepsilon_{d_i}|$.

Therefore, when ϕ goes up, both reaction correspondences shift upwards. However, since the game is now submodular, all we can conclude is that one of the two

المنارة للاستشارات
www.manaraa.com

equilibrium prices must increase (see Amir, 2005). □

Recall from the previous subsection that the elasticity condition $|\varepsilon_{D^i}| \leq |\varepsilon_{d_i}|$ is more compatible with complementary products. As will be seen for the special case of linear demand below, $|\varepsilon_{D^i}| \leq |\varepsilon_{d_i}|$ always holds for complementary while the reverse condition holds for substitutes (under linear demand).

If condition (3.9) is satisfied we can conclude that the price competition is of strategic substitutes and hence the best replies are nonincreasing.[7] In this case we cannot determine unambiguously how the equilibrium prices will react to increased market transparency. From the fact that, as before, (3.7) is negative, whenever $|\varepsilon_{D^i}| > |\varepsilon_{d_i}|$, it follows that both reaction curves shift down, but the latter fact does not imply that both equilibrium prices will necessarily decrease. It is possible that one of them increases if the shifts of the two reaction curves are of unequal magnitudes. Intuitively, this can be explained by the fact that the two effects mentioned before are conflicting now. The direct effect of the downward shift in a firm's reaction curve makes own price go down for each fixed price of the rival, but the indirect effect of adjusting to rival's price moves in the opposite direction, in view of the strategic substitutes property. Thus, the total effect depends on which of these two effects dominates.[8] An explicit closed-form example is provided at the end of the paper to

[7]Here, condition (3.9) is much easier to satisfy for goods that are complements, since D^i_j is then < 0.

[8]For more on the general properties of submodular games, see Amir (1996a), Roy and Sabarwal (2012), and Monaco and Sabarwal (2016). For a recent treatment of the correspondence principle, see Echenique (2002).

المنارة للاستشارات
www.manaraa.com

illustrate the failure of a general result for submodular Bertrand games. Nonetheless, for the special case of symmetric submodular games, we next show that we recover the proposition that both equilibrium prices are decreasing in ϕ for symmetric Bertrand equilibria.

3.3.3 Symmetric games

In the special case of a symmetric duopoly, a definite result on the effects of transparency on price is possible, irrespective of whether prices are strategic substitutes, complements or neither. This subsection deals with this important special case. The only significant restriction is that the result pertains to symmetric (pure-strategy) equilibria, when other (asymmetric equilibria may exist).

Recall that a Bertrand duopoly is symmetric if $P_i = P_j \equiv P$ and $\Pi^i(p_i, p_j) = \Pi^j(p_j, p_i)$. In the following result, the comparative statics result pertains to the symmetric Bertrand equilibrium only, the existence of which requires a quasi-concavity assumption (as the reaction curves may simply have a downward jump that skips over the diagonal). There may exist other, asymmetric equilibria here, and these may well have comparative statics with respect to changes in transparency that do not satisfy the following result.[9]

Proposition 3.5. *Consider a symmetric Bertrand duopoly such that*

(i) $\Pi^i(p_i, p_j)$ *is strictly quasi-concave in own action, and*

(ii) $|\varepsilon_{D^i}| \geq (\leq) |\varepsilon_{d_i}|$, *for all* $(p_i, p_j) \in P_i \times P_j$.

[9] About these asymmetric equilibria, nothing more can be said at this level of generality, beyond what is given in this paper for equilibria of asymmetric games.

www.manaraa.com

Then

(a) a symmetric Bertrand equilibrium exists for all values of ϕ, and

(b) an increase in market transparency ϕ causes the extremal common equilibrium prices of both goods to decrease (increase).

Proof. The strict quasi-concavity of Π^i in p_i guarantees that the reaction curve of player i,

$$r^i(p_j) = \arg\max\{\Pi^i(p_i, p_j) : p_j \in P\}$$

is a continuous single-valued function. It follows that there must exist a symmetric equilibrium, which is not necessarily unique. Consider the extremal symmetric equilibria of the game.

We consider the case $|\varepsilon_{D^i}| \geq |\varepsilon_{d_i}|$. Let $\Pi^i(p_i, p_j, \phi)$ be as defined in the proof of Proposition 3.2. Since $\frac{\partial^2 \log \Pi^i(p_i,p_j,\phi)}{\partial p_i \partial \phi}$ given by (3.7) is negative, we know that $\Pi^i(p_i, p_j, \phi)$ is log-supermodular in (p_i, ϕ). It follows from Topkis's monotonicity theorem (Topkis, 1978) that the reaction curve $r^i(p_j)$ shifts down when ϕ goes up. Invoking the main result in Milgrom and Roberts (1994), we conclude that the extremal symmetric equilibrium prices of the game decrease.

The case $|\varepsilon_{D^i}| \leq |\varepsilon_{d_i}|$ is handled similarly. □

3.3.4 Assumptions on the utility function

We now provide respective equivalent assumptions in terms of the underlying utility function for the elasticity conditions (in both directions) to hold.

المنارة للاستشارات

www.manaraa.com

Proposition 3.6. *Assume demand is derived from a representative consumer according to Option 1. Then*

$$|\varepsilon_{D^i}| > (<)\,|\varepsilon_{d_i}| \ \textit{for all } (p_i, p_j) \in P_i \times P_j$$

if and only if U *satisfies*

$$\frac{U_1(x_1, x_2)}{U_{11}(x_1, x_2) - U_{12}^2(x_1, x_2)/U_{22}(x_1, x_2)} < (>) \frac{U_1(x_1, 0)}{U_{11}(x_1, 0)} \ \textit{for all } (x_1, x_2). \tag{3.10}$$

Proof. Since (D^1, D^2) is the inverse of (P_1^1, P_2^2) in R^2, we can use the inversion relationship to relate the partials of one map to those of the other. Doing so, direct differentiation reveals (upon some computation) that

$$D_1^1 = P_2^2/(P_2^2 P_1^1 - P_1^2 P_2^1) \tag{3.11}$$

Further differentiating (3.2) w.r.t. p_j yields

$$U_{ij}(x_1, x_2) = P_j^i(x_1, x_2), i = 1, 2; j = 1, 2. \tag{3.12}$$

Using (3.11) and (3.12), we have, say for D^1,

$$\varepsilon_{D^1} \triangleq D_1^1 \frac{p_1}{D^1} = \frac{1}{x_1} \frac{U_1(x_1, x_2)}{U_{11}(x_1, x_2) - U_{12}^2(x_1, x_2)/U_{22}(x_1, x_2)}$$

Similarly, starting from (3.4), one arrives at

$$\varepsilon_{d_1} \triangleq d_1' \frac{p_1}{d_1} = \frac{U_1(x_1, 0)}{x_1 U_{11}(x_1, 0)}.$$

Taking into account the standard assumptions on U (see section 2), and keeping in mind that ε_{D^1} and ε_{d_i} are both < 0, the conclusion follows from a direct comparison of $|\varepsilon_{D^1}|$ and $|\varepsilon_{d_1}|$.

A similar argument applies to D^2. $\square$

المنارة للاستشارات
www.manaraa.com

We now provide a general discussion of the scope for condition (3.10) to hold, under both inequalities. We first note that several commonly used utility functions in micro-economics are excluded because the consumption of non-zero amounts of both goods is essential, i.e., these utility functions naturally satisfy the condition that $U(x_1, 0) = U(0, x_2) = 0$. In this case, the uninformed consumers' problem is simply not well defined (and hence neither are the demands d_1 and d_2). A central example of such utility functions is the Cobb-Douglas family $U(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$, with $\alpha > 0, \beta > 0$ and $\alpha + \beta < 1$. This discussion makes it clear that the condition $U(x_1, 0) \neq 0$ and $U(0, x_2) \neq 0$ are necessary for our representative consumer approach to work.

For the standard linear demand for differentiated products (see e.g., Singh and Vives, 1984), it will be seen in the next section that condition (3.10) holds with a "$<$" inequality for substitute goods, and with a "$>$" inequality for complementary goods. This association of the two conditions in (3.10) with the nature of inter-product relationship (i.e., substitutes or complements) appears to be somewhat more general.

As reflected in the results of the last three subsections, the elasticity condition is essentially the critical determinant of the comparative statics of price w.r.t transparency.

Overall, the clear-cut nature of the conclusion of Proposition 3.2 is quite remarkable, given the level of generality of the model, particularly under the broad nature of the interpretation of the model in Option 2. In other words, even when the demands (D^1, D^2) and (d_1, d_2) are totally independent, Proposition 3.2 holds as long

المنارة للاستشارات
www.manaraa.com

as the given elasticity comparison is satisfied.

Next, we discuss the fact that equilibrium selection arguments are needed for the result. As is well-known with supermodular games, in case of multiple equilibria, the comparative statics conclusion would be reversed for those equilibria that are unstable in the sense of best-reply Cournot dynamics. Since the maximal equilibrium (i.e. the one with the highest prices, out of all equilibrium prices) is Pareto dominant for the firms as well as coalition-proof (Milgrom and Roberts, 1996), it is quite a compelling equilibrium for the model at hand. Lastly, the minimal equilibrium is Pareto dominant for consumers, so that it also enjoys a distinguishing property.

The decrease of equilibrium prices can also be interpreted as consisting of two separate effects that push in the same direction. There is a direct effect reflected in the downward shift of the reaction curve as a unilateral reaction of the player to the parameter increase, and an indirect or strategic effect of decreasing own price in response to the decrease in opponent's price, as a consequence of strategic complementarity (see Amir, 2005).

3.4 A linear example

This section contains a numerical example, based on linear demand functions, illustrating some of the main findings of the paper.

Consider a representative informed consumer, as in Option 1, with a standard quadratic utility function given by

$$U(x_i, x_j) + y = a_i x_i + a_j x_j - b_i x_i^2 - b_j x_j^2 - \gamma x_i x_j + y$$

www.manaraa.com

Standard assumptions on the utility function are $U_i(x_i,x_j) = a_i - 2b_i x_i - \gamma x_j > 0, U_{ii}(x_i,x_j) = -2b_i < 0, i = 1,2$ and $U_{ii}(x_i,x_j)U_{jj}(x_i,x_j) - U_{ij}(x_i,x_j)^2 = 4b_ib_j - \gamma^2 > 0$. Maximization of the respective utility functions for the informed and uninformed consumers subject to the budget constraints leads to the following demand systems, for $i = 1,2$:

$$\begin{aligned} D^i(p_1,p_2) &= \frac{2a_ib_j - \gamma a_j}{4b_ib_j - \gamma^2} - \frac{2b_j}{4b_ib_j - \gamma^2}p_i + \frac{\gamma}{4b_ib_j - \gamma^2}p_j \qquad (3.13) \\ d_i(p_i) &= \frac{a_i}{2b_i} - \frac{p_i}{2b_i} \end{aligned}$$

We assume $2a_ib_j - \gamma a_j > 0$ (so the demand function D^i is positive). Moreover, to ensure that each demand reacts more to changes of own price than to changes of the opponent's price, we assume that $|\gamma| < 2b_i, i = 1,2$. The two goods are substitutes if $\gamma > 0$ and complements if $\gamma < 0$.

Firm's i profit is given by:

$$\Pi^i(p_i,p_j) = (p_i - c_i)(\phi(\frac{2a_ib_j - \gamma a_j}{4b_ib_j - \gamma^2} - \frac{2b_j}{4b_ib_j - \gamma^2}p_i + \frac{\gamma}{4b_ib_j - \gamma^2}p_j) + \frac{1-\phi}{2}(\frac{a_i}{2b_i} - \frac{p_i}{2b_i})).$$

Solving for equilibrium prices leads to

$$p_i^* = \frac{(a_i + c_i)\,A_1^2 - 8b_ib_j\phi^2\gamma^2 a_i - 2b_i\phi\gamma A_1\,(a_j - c_j)}{2\,(A_1^2 - 4b_ib_j\gamma^2\phi^2)}, \qquad (3.14)$$

where $A_1 = 4b_ib_j\,(1+\phi) - \gamma^2\,(1-\phi)$. We left to the reader checking that if $\gamma < 0$ then $p_i^* - c_i$ is always positive. This is not necessarily true if $\gamma > 0$. To guarantee that we need to impose additional conditions on parameters. In fact, $D^i(p_i^*, p_j^*) \geq 0$ if and only if

$$\frac{\gamma}{2b_j}\frac{(4b_ib_j - \gamma^2\,(1-\phi))}{(4b_ib_j\,(1+\phi) - \gamma^2)} < \frac{(a_i - c_i)}{(a_j - c_j)}. \qquad (3.15)$$

المنارة للاستشارات
www.manaraa.com

In other words, condition (3.15) says that both firms are active in equilibrium.

Given the linearity of demand, the sign of γ also determines the nature of the strategic interaction between firms. Analyzing the profit function we observe that its cross-partial derivative with respect to (p_i, p_j) is $\Pi^i_{ij}(p_i, p_j) = \frac{\gamma\phi}{4b_ib_j - \gamma^2}$, which has the same sign as γ. Hence, the reaction curves are increasing if goods are gross substitutes and decreasing if they are gross complements. In other words, just as in the standard Bertrand model, prices are strategic complements when goods are substitutes and strategic substitutes when goods are complements. We now show that the sign of γ is also crucial for the elasticity comparison. The proof of the proposition, together with all the other proofs of this section, is presented in the Appendix.

Proposition 3.7. *Consider the demand system for informed and uninformed consumers (3.13).*

(i) If goods are substitutes, the condition $|\varepsilon_{D^i}| \geq |\varepsilon_{d_i}|$ *is satisfied.*

(ii) If goods are complements, the condition $|\varepsilon_{D^i}| \leq |\varepsilon_{d_i}|$ *is satisfied.*

This result implies that if goods are gross substitutes the Bertrand game has strategic complementarities and the elasticity condition $|\varepsilon_{D^i}| > |\varepsilon_{d_i}|$ is satisfied. Hence both equilibrium prices decrease in the transparency level.

In case the goods are gross complements, the game has strategic substitutes and the elasticity condition is satisfied in the opposite direction. Hence, for negative γ, the reaction curves shift up with the level of transparency.

As seen earlier, for the case of strategic substitutes with asymmetric firms, the two prices may move in opposite directions as ϕ changes. This is formalized next.

المنارة للاستشارات
www.manaraa.com

Proposition 3.8. *If $\gamma < 0$ (goods are complements), then either*

- *both equilibrium prices increase in ϕ, or*
- *p_i^* decreases in ϕ while p_j^* increases in ϕ if the following condition is satisfied*

$$-4\gamma\phi A_1 b_j \left(a_i - c_i\right) < \left(a_j - c_j\right)\left(A_1^2 + 4\gamma^2\phi^2 b_i b_j\right). \tag{3.16}$$

Below we provide a numerical example of the situation when one equilibrium price increases and the other one decreases in ϕ.

Example 3.1. Let $P_1 = P_2 = [0, 3]$. Consider the parameter values: $a_1 = 1, a_2 = 2$, $b_1 = 0.5$, $b_2 = 1, \gamma = -.99$, $c_1 = 0.4$, $c_2 = 0.01$, $\phi = 0.5$. All our assumptions are satisfied. The equilibrium prices are then $p_1^* = 0.93$, $p_2^* = 1.03$, and the corresponding profits are respectively $\Pi^1(p_1^*, p_2^*) = 0.34$ and $\Pi^2(p_1^*, p_2^*) = 0.77$.When the transparency level ϕ increases to 0.8, the best responses of both firms shift upward. They cross at the new equilibrium prices $\hat{p}_1^* = 0.89$, $\hat{p}_2^* = 1.04$. Clearly, p_2 has increased, but p_1 has decreased as ϕ went from 0.5 to 0.8.

With the assumed linear form of the demand we can study, how the transparency level influences firms' outputs, profits and social welfare in equilibrium.

First let us study the impact of increase in transparency on equilibrium output of a single firm, say i. To do this, we note that this impact can be express as a sum of two effects: the demand effect $D^i(p_i, p_j) - \frac{1}{2}d_i(p_i)$ and the indirect effect $p_i^{*\prime}\left(\phi D_i^i(p_i^*, p_j^*) + \frac{1-\phi}{2}d_i'(p_i^*)\right) + \phi D_j^i(p_i^*, p_j^*)p_j^{*\prime}$. We have to check if this effects are positive or negative.

www.manaraa.com

In case of complements ($\gamma < 0$), the demand effect is always positive. Also for substitutes (or $\gamma > 0$), for the symmetric case ($a_1 = a_2$ and $b_1 = b_2$), it can be shown that the demand effect is positive (this is the case in Boone and Potters, 2002).

The example below illustrates that the demand effect can be negative for one firm in the asymmetric case with substitutes.

Example 3.2. Consider the following parameter values: $a_1 = 1$, $b_1 = 1$, $\gamma = 0.9$, $c_1 = 0.1$, $a_2 = 1.9$, $b_2 = 1$, $c_2 = 0.01$, $\phi = 0.5$ and let $P_1 = P_2 = [0, 1]$. The equilibrium prices are $p_1^* = 0.38, p_2^* = 0.86$. The demand effects are given by

$$\frac{2a_ib_j - \gamma a_j - 2b_jp_i + \gamma p_j}{4b_ib_j - \gamma^2} - \frac{1}{2}\frac{a_i - p_i}{2b_i}$$
$$= (a_i - p_i)\left(\frac{2b_j}{4b_ib_j - \gamma^2} - \frac{1}{4b_i}\right) - (a_j - p_j)\frac{\gamma}{4b_ib_j - \gamma^2}$$

For firm 1, this is -0.09, but for firm 2 this effect is positive and equal to 0.23.

Nevertheless, the sum of the demand effects is always positive. This result is formulated as a lemma, since it is useful in the proofs of further results.

Lemma 3.9. *The sum of the demand effects is always positive.*

Apart from the demand effect, the equilibrium output derivative with respect to ϕ contains also the already mentioned second part, which can be called an indirect effect since it measures an influence of changes in equilibrium prices on the total output of firm i.

Proposition 3.10. *The indirect effect of transparency on a firm's equilibrium output is positive if and only if $\gamma > 0$ and negative if and only if $\gamma < 0$.*

المنارة للاستشارات
www.manaraa.com

This result is not enough to establish the effect on equilibrium output of an increase in ϕ, since in case of substitute goods we are not sure about the demand effect sign, and in case of complements the two effects, demand and indirect, work in opposite directions. However the following holds.

Proposition 3.11. *If $\gamma < 0$ the equilibrium output of firm i is increasing in ϕ.*

If $\gamma > 0$, the equilibrium output of firm i does not necessarily increase with ϕ. In this case the indirect effect is always positive but the demand effect can be negative and outweigh the positive one. This is the case in Example 3.2, where for firm 1 the equilibrium output derivative with respect to ϕ, is -0.00378. However, the effect on the sum of outputs of both firms is clear.

Proposition 3.12. *The total equilibrium output (the sum of the equilibrium outputs of the two firms) is increasing in ϕ.*

In case of firms' profits, the impact of the transparency increase can be ambiguous. If the goods are complements, the effect on profit is always positive, even though one of the prices may decrease. In case of substitutes, we know that both prices go down, so one could expect this is not beneficial for firms. But larger transparency may lead to an output expansion which can outweigh the profit loss connected with the price decrease. This intuition is formalized next.

Proposition 3.13. *$\gamma < 0$ implies the equilibrium profit $\Pi^i(p_1^*(\phi), p_2^*(\phi))$ is increasing in ϕ. If the demand effect of firm i is negative the equilibrium profit $\Pi^i(p_1^*(\phi), p_2^*(\phi))$ is decreasing in ϕ.*

المنارة للاستشارات
www.manaraa.com

Finally, we can show the following result on the effects on welfare.

Proposition 3.14. *Social welfare is increasing in ϕ.*

3.5 Conclusion

This paper has analyzed the effects of market transparency in the context of a general formulation of a differentiated-goods Bertrand price competition model. In the more standard case of strategic complementarity of prices, we directly generalize the results of Schultz (2004) for the Hotelling model in terms of equilibrium prices and consumer surplus. While prices are decreasing in the transparency level and consumers are better off, firms are necessarily worse off only in the case of substitute goods. Otherwise, one of them may gain, and even, in some cases of complementary goods, both profits may increase.

In case of strategic substitutability of prices, the results are ambiguous even for equilibrium prices. Indeed, one of the prices may well increase with the level of transparency, as established via an example. This is in line with well-known results on the comparative statics properties of submodular games. The lack of a clear-cut result on price changes precludes general conclusions about profits and consumer surplus. However, when both prices decrease with transparency, consumers gain and in case of complementary goods, both firms gain as well.

We provide a thorough investigation of the properties of the model for the important special case where the demand function is the standard linear demand for differentiated products (e.g., Singh and Vives, 1984). This characterization includes

www.manaraa.com

clear-cut results on the effects of transparency on prices, outputs, profits and social welfare. This provides the Bertrand counterpart to the Cournot case (with linear demand) analyzed by Boone and Potters (2006).

3.6 Appendix

This section contains all the proofs of Section 3.4.

Proof of Proposition 3.7. The demand elasticity (in absolute value) of informed consumers is

$$|\varepsilon_{D^i}| = \left|\frac{p_i}{D^i}D_i^i\right| = \left|-\left(\frac{2a_ib_j-\gamma a_j}{4b_ib_j-\gamma^2}-\frac{2b_j}{4b_ib_j-\gamma^2}p_i+\frac{\gamma}{4b_ib_j-\gamma^2}p_j\right)^{-1}p_i\frac{2b_j}{4b_ib_j-\gamma^2}\right|$$

The absolute value of the elasticity of the uniformed consumers is given by

$$|\varepsilon_{d_i}| = \left|\frac{p_i}{d_i}d_i'\right| = \left|-\left(\frac{a_i}{2b_i}-\frac{p_i}{2b_i}\right)^{-1}\frac{p_i}{2b_i}\right|$$

Setting $|\varepsilon_{D^i}| \geq |\varepsilon_{d_i}|$ leads (after computations) to the condition $0 \leq \gamma\left(a_j - p_j\right)$, or $\gamma \geq 0$. $\square$

Proof of Proposition 3.8. When $\gamma < 0$ the imperfect transparency game is of strategic substitutes. From Propositions 3.2 and 3.7 follows that an increase in the transparency level makes both reaction curves to shift up, hence the equilibrium prices cannot both decrease.

A necessary condition for one of the equilibrium prices, say i, to decrease, is

$$\frac{1}{2b_i}\left[a_i-2p_i^*+c_i\right]\left[-2\left(\phi\frac{2b_i}{4b_ib_j-\gamma^2}+\frac{1-\phi}{2}\frac{1}{2b_j}\right)\right]-\frac{\phi}{2b_j}\left[a_j-2p_j^*+c_j\right]\frac{\gamma}{4b_ib_j-\gamma^2}>0$$

www.manaraa.com

Reordering yields after simplification $-[a_i - 2p_i^* + c_i]\, A_1 > 2b_i\phi\gamma[a_j - 2p_j^* + c_j]$. Using the equilibrium prices (3.14) we obtain condition (3.16) after some computations. $\square$

Proof of Lemma 3.9. The sum of the demand effects $\sum_{i=1}^{2}\left(D^i - \frac{1}{2}d_i\right)$ is $\frac{K}{8b_ib_j(4b_ib_j-\gamma^2)} \times$ $\left[(a_i - c_i)\, b_j\, (A_2 - 2b_i\gamma(A_1 + (4b_ib_j - \gamma^2))) - (a_j - c_j)\, b_i\, (2b_j\gamma(A_1 + (4b_ib_j - \gamma^2)) - A_2)\right]$, where A_1 is as defined in Section 3.4, $A_2 = (4b_ib_j - z^2)(4b_ib_j + z^2) + \phi\left(z^4 + 16b_i^2b_j^2\right) > 0$, and

$K = \frac{A_1}{A_1^2 - 4b_ib_jz^2\phi^2} > 0$ as

$$A_1^2 - 4b_ib_jz^2\phi^2 = \left(4b_ib_j - z^2\right)^2 + 2\phi\left(4b_ib_j + z^2\right)\left(4b_ib_j - z^2 + 2\phi b_ib_j\right) + z^4\phi^2 > 0$$

We wish to show that

$$\begin{aligned} &(a_i - c_i)\, b_j \left(A_2 - 2b_i\gamma(A_1 + \left(4b_ib_j - \gamma^2\right))\right) \\ &- (a_j - c_j)\, b_i \left(2b_j\gamma(A_1 + \left(4b_ib_j - \gamma^2\right)) - A_2\right) > 0. \end{aligned} \tag{3.17}$$

There are four possible cases to be considered:

1. $A_2 - 2b_i\gamma(A_1 + (4b_ib_j - \gamma^2)) > 0$ and $(2b_j\gamma(A_1 + (4b_ib_j - \gamma^2)) - A_2) > 0$
2. $A_2 - 2b_i\gamma(A_1 + (4b_ib_j - \gamma^2)) < 0$ and $(2b_j\gamma(A_1 + (4b_ib_j - \gamma^2)) - A_2) > 0$
3. $A_2 - 2b_i\gamma(A_1 + (4b_ib_j - \gamma^2)) > 0$ and $(2b_j\gamma(A_1 + (4b_ib_j - \gamma^2)) - A_2) < 0$
4. $A_2 - 2b_i\gamma(A_1 + (4b_ib_j - \gamma^2)) < 0$ and $(2b_j\gamma(A_1 + (4b_ib_j - \gamma^2)) - A_2) < 0$

Note that in case of $\gamma < 0$ the only possibility is case 3, and in this case (3.17) is naturally satisfied. Hence, we consider $\gamma > 0$ and we would like to show that (3.17) is satisfied also in cases 1 and 4 and case 2 is not possible, given our assumptions.

المنارة للاستشارات
www.manaraa.com

Case 1 Using (3.15) for firm i we can replace $(a_j - c_j)$ in the left hand side (henceforth LHS) of (3.17) to obtain a smaller expression. After rearranging this expression we obtain $(4b_ib_j - \gamma^2)^2 + 2(4b_ib_j - \gamma^2)(4b_ib_j + \gamma^2)\phi + \left((4b_ib_j + \gamma^2)^2 - 4\gamma^2 b_ib_j\right)\phi^2$ which is positive, given our initial assumptions. Hence, we conclude that (3.17) holds.

Case 4 As before we use (3.15), but this time for firm j and we can replace $(a_i - c_i)$ in the LHS of (3.17) to obtain a smaller expression. After rearranging this expression we obtain, the same as in case 1, $(4b_ib_j - \gamma^2)^2 + 2(4b_ib_j - \gamma^2)(4b_ib_j + \gamma^2)$ $\phi + \left((4b_ib_j + \gamma^2)^2 - 4\gamma^2 b_ib_j\right)\phi^2$ which is positive, given our initial assumptions. Hence, we conclude that (3.17) holds.

Case 2 Suppose $A_2 - 2b_i\gamma(A_1 + (4b_ib_j - \gamma^2)) < 0$ and $(2b_j\gamma(A_1 + (4b_ib_j - \gamma^2)) - A_2) > 0$, then $\gamma(b_i + b_j)(A_1 + 4b_ib_j - \gamma^2) > A_2$. Using definitions of A_1 and A_2 this can be rewritten as

$$\gamma^4 - \gamma^3(b_i + b_j) - 4\gamma b_ib_j(b_i + b) + 16b_i^2b_j^2 < -\frac{1}{\phi}(2b_j - \gamma)(2b_i - \gamma)\left(4b_ib_j - \gamma^2\right). \tag{3.18}$$

The right hand side (henceforth RHS) is negative. We want to show that the LHS is positive, hence there is a contradiction.

Assume w.l.o.g that $b_j < b_i$ and replace in the LHS γ by $2b_j$. This way we obtain $(2b_j)^4 - (b_i + b_j)(2b_j)^3 - 4b_ib_j(b_i + b_j)2b_j + 16b_j^2b_i^2 = 8b_j^2(b_i - b_j)^2 > 0$. Now we show that

$$\gamma^4 - \gamma^3(b_i + b_j) - 4\gamma b_ib_j(b_i + b) + 16b_i^2b_j^2 \tag{3.19}$$

www.manaraa.com

$$> (2b_j)^4 - (b_i + b_j)(2b_j)^3 - 4b_ib_j(b_i + b_j)2b_j + 16b_j^2b_i^2$$

since this means that the LHS of (3.18) is positive.

To do this we compute the difference of the left and the right hand side of (3.19). This is positive if and only if

$$(b_i - b_j)\gamma^2 - \gamma^3 + 2b_j(b_i - b_j)\gamma + 4b_j\left(b_i^2 - b_j^2 + 2b_ib_j\right) > 0.$$

To show this we add and subtract an additional term $4b_ib_j\gamma$ and obtain

$$(b_i - b_j)\gamma^2 + 4b_ib_j\gamma - \gamma^3 + 2b_j(b_i - b_j)\gamma + 4b_j\left(b_i^2 - b_j^2 + 2b_ib_j\right) - 4b_ib_j\gamma = (b_i - b_j)\gamma^2$$

$$+\left(4b_ib_j - \gamma^2\right)\gamma + 2b_j(b_i - b_j)\gamma +$$

$4b_j\left(b_i(2b_j - \gamma) + b_i^2 - b_j^2\right) This is positive since all the components are positive.$

Summarizing, we have shown that the LHS of (3.18) is positive, hence it cannot be less than the RHS, which is negative. That is why $A_2 - 2b_i\gamma(A_1 + (4b_ib_j - \gamma^2)) <$ 0 and $2b_j\gamma(A_1 + (4b_ib_j - \gamma^2)) - A_2 > 0$ is a contradiction. □

Proposition 3.10. The indirect effect on firm i's equilibrium output is given by

$$[A_1A_3(a_j - c_j) - 16b_j^2b_i\phi^3\gamma^3(a_i - c_i)]\gamma/4A_4^2$$

where A_1 is as defined in Section 3.4,

$A_3 = \phi^2\left(\gamma^4 + 4b_ib_j\left(4b_ib_j - \gamma^2\right)\right) + 2\phi\left(4b_ib_j - \gamma^2\right)\left(4b_ib_j + \gamma^2\right) + \left(4b_ib_j - \gamma^2\right)^2 > 0$, and

$A_4 = \phi^2\left(\left(4b_ib_j + \gamma^2\right)^2 - 4\gamma^2b_ib_j\right) + 2\phi\left(4b_ib_j - \gamma^2\right)\left(4b_ib_j + \gamma^2\right) + \left(4b_ib_j - \gamma^2\right)^2 > 0.$

المنارة للاستشارات
www.manaraa.com

If $\gamma < 0$, the indirect effect is negative. For the case $\gamma > 0$, observe that the sign of the indirect effect is the same as the sign of the numerator. Assume it is negative, then

$$\frac{A_1 A_3}{8b_j b_i \phi^3 \gamma^3} < 2b_j \frac{(a_i - c_i)}{(a_j - c_j)} \tag{3.20}$$

Conditions (3.20) and (3.15) are in contradiction since $\frac{A_1 A_3}{8b_j b_i \phi^3 \gamma^3} > \frac{\gamma\left(4b_i b_j - \gamma^2(1-\phi)\right)}{(4b_i b_j (1+\phi) - \gamma^2)}$. Therefore the indirect effect of a ϕ increase on total equilibrium output of a firm is positive if $\gamma > 0$. □

Proposition 3.11. This is based on the same idea as above, namely one can check (after some computations) that the demand effect overweights the indirect effect when both firms are active in the market in equilibrium (computational details available upon request). □

Proposition 3.12. Let $\gamma > 0$. From Lemma 3.9 the sum of the demand effects is positive and from Proposition 3.10, the indirect effects are positive as well. If $\gamma < 0$ we conclude by Proposition 3.11 that both firms outputs increase when ϕ increases. □

Proposition 3.13. The derivative of equilibrium profit with respect to ϕ is given by

$$\begin{aligned} \frac{\partial}{\partial \phi} \Pi^i(p_1^*, p_2^*) &= (p_i^* - c_i) \left(D^i - \frac{1}{2} d_i + \phi D_j^i p_j^{*\prime}(\phi) \right) \\ &= (p_i^* - c_i) \frac{A_5 (a_i - c_i) - 2b_i \gamma (a_j - c_j) A_6}{8A_7^2 (4b_j b_i - \gamma^2) b_i}, \end{aligned}$$

المنارة للاستشارات
www.manaraa.com

where

$$
\begin{aligned}
A_5 &= \left(4b_ib_j+\gamma^2\right)\left(4b_ib_j-\gamma^2\right)^4 \\
&\quad +7\phi^4\left(4b_ib_j+\gamma^2\right)\left(\gamma^4+16b_i^2b_j^2\right)\left(\left(4b_ib_j+\gamma^2\right)^2-4\gamma^2b_ib_j\right) \\
&\quad +4\phi\left(\left(4b_ib_j+\gamma^2\right)^2-4\gamma^2b_ib_j\right)\left(4b_ib_j-\gamma^2\right)^3 \\
&\quad +4\phi^3\left(4b_ib_j-\gamma^2\right)\left(\gamma^4-2\gamma^2b_ib_j+16b_i^2b_j^2\right)\left(4b_ib_j+\gamma^2\right)^2 \\
&\quad +2\phi^2\left(4b_ib_j+\gamma^2\right)\left(3\gamma^4+2\gamma^2b_ib_j+48b_i^2b_j^2\right)\left(4b_ib_j-\gamma^2\right)^2>0,
\end{aligned}
$$

$$
\begin{aligned}
A_6 &= \left(4b_ib_j\left(1+\phi\right)-\gamma^2\left(1-\phi\right)\right)2\left(4b_ib_j-\gamma^2\right)^3 \\
&\quad +\phi^3\left(4b_ib_j+\gamma^2\right)\left(\left(4b_ib_j+\gamma^2\right)^2-4\gamma^2b_ib_j\right) \\
&\quad +5\phi\left(4b_ib_j+\gamma^2\right)\left(4b_ib_j-\gamma^2\right)^2+4\phi^2\left(2b_ib_j+\gamma^2\right)\left(8b_ib_j+\gamma^2\right)\left(4b_ib_j-\gamma^2\right)>0,
\end{aligned}
$$

$$A_7=-\left(\left(4b_ib_j+\gamma^2\right)^2-4\gamma^2b_ib_j\right)\phi^2-2\left(4b_ib_j-\gamma^2\right)\left(\gamma^2+4b_ib_j\right)\phi-\left(4b_ib_j-\gamma^2\right)^2<0.$$

$\frac{\partial}{\partial\phi}\Pi^i(p_1^*,p_2^*)$ is positive if $\gamma<0$, or when the goods are complements. If $\gamma>0$, $D_j^i>0$ and $p_j^{*\prime}(\phi)<0$, hence a negative demand effect is enough for $\frac{\partial}{\partial\phi}\Pi^i(p_i^*,p_j^*)<0$. □

Proposition 3.14. We have $\frac{d}{d\phi}W(p_1^*(\phi),p_2^*(\phi),\phi)=T_1+T_2$, where

$$T_1=(p_i^*-c_i)\left[\phi(D_i^ip_i^{*\prime}+D_j^ip_j^{*\prime})+\frac{1-\phi}{2}d_i'p_i^{*\prime}\right]+(p_j^*-c_j)\left[\phi(D_i^jp_i^{*\prime}+D_j^jp_j^{*\prime})+\frac{1-\phi}{2}d_j'p_j^{*\prime}\right]$$

and

$$T_2=U(D^i,D^j)-c_iD^i-c_jD^j-\frac{1}{2}[U(d_i,0)-c_id_i]-\frac{1}{2}[U(0,d_j)-c_jd_j].$$

www.manaraa.com

T_1 consists of the indirect effects of ϕ on total equilibrium outputs of the firms, weighted by their margins. Hence, when $\gamma > 0$, it is positive and when $\gamma < 0$, it is negative.

T_2 is always positive (all necessary computations can be provided upon request), and to show this we need to use the fact that (3.15) must hold for both firms.

Hence, if $\gamma > 0$, $T_1 + T_2 > 0$. If $\gamma < 0$, $T_1 + T_2 > 0$ as well, but showing this requires direct computation and condition (3.15) for both firms (computational details upon request). $\square$

المنارة للاستشارات

www.manaraa.com

CHAPTER 4

EQUAL TREATMENT AND SOCIALLY OPTIMAL R&D IN DUOPOLY WITH ONE-WAY SPILLOVERS

Based on joint work with Giuseppe De Feo and Malgorzata Knauff.

4.1 Introduction

In the context of non-tournament models of research and development (henceforth, R&D) in which firms engage in cost-reducing innovation and then compete la Cournot in the product market, it is widely recognized that exogenous knowledge spillovers create distortions in R&D investment decisions (see e.g., the pioneering study by Spence, 1984). Along with other distortions associated with such models, such as imperfect competition, these spillovers cause a well-known significant wedge between the private and the public returns to R&D, leading to insufficient levels of R&D being supplied from the perspective of social optimality (see e.g., Bernstein and Nadiri, 1988).

The bulk of the extant literature on imperfectly appropriable R&D focuses on deterministic multidirectional spillovers.[1] A fixed proportion (given by the spillover parameter) of every firm's R&D effort or benefit flows freely to the rivals. As argued by Kamien et al. (1992), such a spillover process is appropriate if the associated

[1] A small selection of papers includes Ruff (1969), Katz (1986), d'Aspremont and Jacquemin (1988, 1990), Kamien et al. (1992) and Amir (2000), among many others.

المنارة للاستشارات
www.manaraa.com

R&D process in the extant literature is implicitly assumed to be a "multidimensional heuristic rather than a one-dimensional algorithmic process." Thus, it necessarily involves trial and error on the part of the firms, which follow potentially different sets of research paths and or approaches.

An exception to the deterministic multidirectional spillovers is proposed in the two studies by Amir and Wooders (1999, 2000), henceforth AW. These authors consider instead a stochastic directed spillover process whereby know-how may flow only from the more R&D intensive firm to its rival[2]. In their model, spillovers are stochastic and admit only extreme realizations - either full or no spillovers occur with a given probability[3]. The latter probability is itself defined as the spillover parameter. As argued by AW, the idea underlying the assumption of a uni-directional spillover process is that it is a better approximation for the potential leakages that occur when the R&D process is either one-dimensional, i.e. there is a single research path to achieve unit cost reductions, or multi-dimensional in which case this spillover structure suggests that there is a well-defined natural path to follow. In this context, the spillover parameter may be interpreted as being related to the length of patent protection, but also to a measure of the imitation lag.

The purpose of the present paper is to consider a deterministic one-way spillover

[2]That spillovers are an important aspect of firms' overall business strategy is well-documented (see e.g., Billand et al. (2016) for an overview of the related literature). In addition, there are multiple channels through which spillovers might flow, including in trade-related contexts (see e.g., Ferrier et. al, 2016).

[3]For related settings, see also Jin and Troege (2006), Hinloopen (1997, 2000), Martin (2002), and Tesoriere (2008).

المنارة للاستشارات
www.manaraa.com

process, which constitutes the certainty-equivalent version of AW's model. In other words, a fraction of the R&D lead of the leader (i.e., of the R&D differential between the two firms) flows to its rival with certainty. That fraction is itself defined as the spillover parameter, and ranges from zero (when R&D is a pure private good) to one (when R&D is a pure public good).

As to the rest of the model, we consider the standard two-period model of process R&D and product market competition with the said deterministic one-way spillover process. We adopt the common specification of linear market demand and identical linear firms' cost functions to facilitate closed-form solutions and comparability of outcomes with past literature.

We now give an overview of the main results of the paper and some general discussion. Though firms are ex-ante identical, one obtains a unique pair of asymmetric equilibria so that the roles of R&D innovator (the more R&D intensive firm) and imitator (the less R&D intensive firm) are endogenously determined. That is, a firm always either spends less than its rival so as to free ride on the latter's R&D investment through spillovers, or spends more if the other firm's investment is too low in order to benefit from a competitive advantage over its rival in the product market. This outcome produces asymmetries in terms of the unit cost structure in the product market competition, and thus unequal market shares. This conclusion establishes a simple link between the nature of the R&D process in an industry - including the associated spillover - and the emergence of inter-firm heterogeneity in that industry.

As we shall see below, it turns out to be more convenient to examine some of

المنارة للاستشارات
www.manaraa.com

the economic issues considered in this paper with a deterministic spillover process than with its stochastic analog (proposed by AW). One aspect of this choice is motivated by the ease of comparison with the deterministic multi-way spillover processes typically used in the literature (as in d'Aspremont and Jacquemin, 1988, Kamien, Muller and Zang, 1992, and Amir, 2000). Another difference between this setting and the stochastic version in AW is that endogenous heterogeneity of firms in terms of R&D and final unit costs holds with certainty in the present setting, but only with positive probability in the AW model.

Kamien et al. (1992) have shown that, when the spillover process is multidirectional and deterministic, cooperating through a joint lab, thereby allowing firms to jointly appropriate the outcome of R&D investments, while sharing the associated costs equally, is superior to R&D competition in terms of levels of investments, industry profit and consumer surplus.[4]. In the context of one-way stochastic spillovers, AW find that, under R&D competition, the innovator sometimes invests more in R&D than the joint lab, and the industry's total profit is sometimes higher than under the joint lab. Clearly, since spillovers vanish under this type of cooperation, the same results obtain with deterministic one-way R&D spillovers.[5]

In the most important part of the paper, we consider a benevolent central

[4]More precisely, these authors considered a cartelized joint venture defined as an R&D cartel (i.e, firms choose R&D levels to maximize their total Cournot profits) wherein firms internally set the spillover parameter to its maximal value of 1. Amir (2000) shows that this cooperation scenario is equivalent to a joint lab.

[5]The literature on R&D cooperation has more recently been extended to other areas of economics, including environmental innovation (McDonald and Poyago-Theotoky, 2016), and the organization of the firm (Chalioti, 2015).

www.manaraa.com

planner with a second best mandate, i.e. one that can decide on R&D investments without intervening as far as market conduct is concerned (as in Suzumura, 1992). We consider two different scenarios, one in which the planner is subject to the political constraint of equal treatment of the firms, and one in which the planner is free from such contraints. While the second-best optimal symmetric investments coincide with those of the joint lab in the first scenario, social welfare achieved under the joint lab is superior since R&D costs are shared among firms. Therefore, the joint lab emerges as a desirable way to implement a contrained second-best optimal scenario without actual intervention by a social planner (and with built-in avoidance of R&D duplication costs).

Furthermore, since imposing symmetric R&D investments yields symmetric final unit costs in the product market competition, social welfare under R&D competition may dominate that under the symmetry-constrained central planner; in fact, this happens when R&D costs are low enough. Intuitively, this is not that surprising since social welfare tends to be higher when firms are asymmetric in terms of unit costs (see Salant and Shaffer, 1998, 1999). In fact, as the latter studies brought to the fore, welfare maximization often entails endogenous discriminatory treatment of firms even under the standard multi-directional spillover structure. Thus, one important motivation for the welfare part of the present paper is precisely that this endogenous discriminatory outcome will even be more significant under a uni-directional spillover structure.

In this respect, it is obviously of interest to get a handle on the extent of

www.manaraa.com

welfare loss incurred by society as a result of the politically-motivated constraint of equal treatment of firms in regulation. Relaxing the assumption that the central planner imposes equal treatment of firms, we find that social welfare induced by the second-best welfare maximizing asymmetric R&D investments dominate that of the joint lab if either the spillover parameter or the cost of performing R&D are low enough. Therefore, the well known result that the market typically delivers lower levels of R&D than a (second-best) social planner continues to hold in our setting, despite the resulting asymmetry among firms[6]. Finally, we show that the efficiency loss due to equal treatment increases with the size of the spillover parameter, and may amount to a maximal level of about 45% in relative terms. We argue that this is a surprisingly high and significant loss, and that, broadly speaking, market regulators may be well-advised to take this into account when conceiving regulatory schemes.

The rest of the paper is organized as follows. Section 4.2 describes the basic non-cooperative R&D model and the associated assumptions. Section 4.3 characterizes the equilibrium under R&D competition. R&D cooperation by means of a joint lab is considered in Section 4.4. An extensive second-best welfare analysis is provided in Section 4.5. Concluding remarks are provided in Section 4.6. All the proofs (in the form of brief calculations) are provided in Section 4.7.

[6] For instance, Burr et al. (2013) provides some insight into the well-known wedge between private and social levels of R&D. See also Stepanova and Tesoriere (2011).

www.manaraa.com

4.2 The model

The basic model is a standard two-stage duopoly consisting of a process R&D choice in the first stage and subsequent Cournot competition in the second stage in the tradition of the literature following Katz (1984) and d'Aspremont and Jacquemin (1988). However, while R&D is still subject to involontary spillovers, these will be taken to one-way or uni-directional in the present study, following Amir and Wooders (1999), henceforth AW.

Formally, consider an industry with two firms producing a homogenous good with the same initial unit cost c, playing the following two-stage game. In the first stage, firms simultaneously choose their autonomous cost reduction level x_1 and x_2, with $x_i \in [0, c]$, $i = 1, 2$. The R&D cost to firm i associated with the cost reduction x_i is

$$C(x_i) = \frac{\gamma}{2} x_i^2, i = 1, 2.$$

We assume, following AW, that spillovers are uni-directional, or in other words that know-how flows only from the more R&D intensive firm (called the innovator) to its rival (the imitator). However, contrary to AW, we assume that the spillover process is deterministic. Namely, if autonomous cost reductions are x_1 and x_2 with, say, $x_1 \geq x_2$, then the effective or final cost reductions are

$$X_1 = x_1 \quad \text{and} \quad X_2 = x_2 + \beta \left(x_1 - x_2 \right),$$

where the parameter $\beta \in [0, 1]$, called the spillover parameter, is the fraction of the

www.manaraa.com

difference in cost reductions that spills over to firm 2 (with certainty).[7] Thus the imitator ends up with its own cost reduction plus a fraction of the innovator's lead. This is a natural definition of spillovers in settings where the R&D process is one-dimensional, reflecting in particular that firm 1 has nothing to possibly learn from firm 2.

This deterministic spillover process may be seen as the certainty-equivalent version of the stochastic spillover process introduced by AW. Both spillover processes are a reflection of the R&D process itself being a one-dimensional process, i.e., a well-defined sequence of hurdles or tests that firms may pursue in their search for discovery.[8]

In the second stage, upon observing the new unit costs, firms compete in the product market by choosing quantities, facing a linear inverse demand

$$P(q_1 + q_2) = a - (q_1 + q_2).$$

A pure strategy for firm i is thus a pair (x_i, q_i), where $x_i \in [0, c]$ and $q_i : [0, c]^2 \to \mathbb{R}_+$. Throughout, we use the standard concept of subgame perfect equilibrium.

We assume that demand is high enough relative to the initial unit cost to ensure that the second-stage game admits a unique pure strategy Nash equilibrium

[7] An analogous pair of expressions for the final cost reductions holds for the case $x_1 \leq x_2$, and is thus omitted.

[8] More precisely, the process need not be uni-dimensional as long as there is a natural sequence of search steps that all firms would undertake. This is quite distinct from the multi-dimensional heuristic proposed by Kamien et. al. (1992) as an appropriate R&D process corresponding to the multi-way spillover process that is widely adopted in the literature, starting with Spence (1984). Naturally, different industries will be better approximated by one or the other of these two categories of spillover process.

المنارة للاستشارات
www.manaraa.com

(PSNE) where both firms are active in the product market for all possible R&D levels that they may undertake, that is,

(A1) $a > 2c$

Cournot equilibrium profit of firm i in the second stage, given the actual unit costs c_i, c_j, is thus given by $\Pi(c_i, c_j) = (a - 2c_i + c_j)^2/9$. Firms' *net* profits F_1, F_2, defined as the difference between the second stage profit and the first stage R&D investment, can then be expressed as functions of the autonomous cost reductions x_1 and x_2. Since the game is symmetric, we have that $F_1(x_1, x_2) = F_2(x_2, x_1)$. Therefore, throughout the paper, we omit the subscripts and write $F(x_i, x_j)$ to denote the net profit of firm i, where

$$F(x_i, x_j) = \begin{cases} \frac{1}{9}[a - c + x_i\,(2-\beta) - x_j\,(1-\beta)]^2 - \frac{\gamma}{2}x_i^2 & \hat{=} U(x_i, x_j) \quad \text{if } x_i \geq x_j \\ \frac{1}{9}[a - c + 2x_i\,(1-\beta) + x_j\,(2\beta - 1)]^2 - \frac{\gamma}{2}x_i^2 & \hat{=} L(x_i, x_j) \quad \text{if } x_i \leq x_j \end{cases} \tag{4.1}$$

One can easily check that F is globally continuous, concave in the two triangles above and below the diagonal, but has concavity-destroying kink along the diagonal. Furthermore, for $\beta \leq \frac{1}{2}$, both U and L are submodular in (x_i, x_j), i.e., $\frac{\partial^2 U(x_i,x_j)}{\partial x_i \partial x_j} < 0$ and $\frac{\partial^2 L(x_i,x_j)}{\partial x_i \partial x_j} < 0$. On the other hand, for $\beta > \frac{1}{2}$, U is submodular but L is supermodular in (x_i, x_j), i.e., $\frac{\partial^2 U(x_i,x_j)}{\partial x_i \partial x_j} < 0$ and $\frac{\partial^2 L(x_i,x_j)}{\partial x_i \partial x_j} > 0$.

Furthermore, we assume the following:

(A2) $9\gamma > 2\,(2-\beta)^2$

(A3) $9\gamma > 4\frac{a}{c}\,(1-\beta)$

المنارة للاستشارات
www.manaraa.com

Close variants of these assumptions are quite standard in the R&D literature. (A2) guarantees that U and L are strictly concave with respect to own R&D level, and may thus be thought of as a global second-order condition. (A3) ensures that firm i's reaction function is interior, or that it satisfies $r_i(c) < c$, where $r_i(x_j) \in \text{argmax}\{F(x_i, x_j) : x_i \in [0, c]\}$.

4.3 The non-cooperative equilibrium

In this section, we analyze the subgame-perfect equilibria of the two-stage game, to be referred to as Case N (for non-cooperative scenario). Equivalently, we analyze the Nash equilibria of the game in R&D choices, given the unique Cournot equilibrium in the second stage (with payoffs given by (4.1)).

Under (A2)-(A3), one can derive the reaction function of, say, firm i as

$$r_i(x_j) = \begin{cases} 2\frac{(2-\beta)(a-c+x_j(\beta-1))}{9\gamma-2(\beta-2)^2} & \text{if } x_i \geq x_j \\ 4\frac{(1-\beta)(a-c+x_j(2\beta-1))}{9\gamma-8(\beta-1)^2} & \text{if } x_i \leq x_j \end{cases} \tag{4.2}$$

and since the game is symmetric, we have $r_i(x_j) = r_j(x_i)$.

Before characterizing the equilibrium investments of the first-stage R&D game, our first result sheds light on a key feature of the model, that firms' reaction functions cannot be continuous.

Lemma 4.1. *The reaction functions admit a unique downward jump that skips over the* $45°$ *line.*

Figure 1a (resp. 1b) depicts firms' reaction curves for $\beta \leq \frac{1}{2}$ (resp. $\beta > \frac{1}{2}$). As was previously mentionned, the upper payoff function U is globally submodular

www.manaraa.com

in own and rival's decisions so that it gives rise to a reaction function segment that shifts down as rival's investment increases. As for the lower payoff function L, it is also submodular in own and rival's decision for $\beta \leq \frac{1}{2}$ (thus reflecting strategic substitutes), but supermodular for $\beta > \frac{1}{2}$, so that its reaction function segment shifts up (thus reflecting strategic complements) as rival's investment increases for this range of the spillover parameter.[9]

Given the firms' reaction functions, straightforward computations establish that reaction curves cross at $(\overline{x}, \underline{x})$ and $(\underline{x}, \overline{x})$, where

$$\begin{cases} \bar{x} = & \frac{1}{D_N} 2(2-\beta)\left[3\gamma - 4(1-\beta)^2\right](a-c) \text{ and} \\ \underline{x} = & \frac{1}{D_N} 4(1-\beta)\left[3\gamma - 2(1-\beta)(2-\beta)\right](a-c) \end{cases}$$

where

$$D_N \triangleq 27\gamma^2 - 6\gamma\left(5\beta^2 - 12\beta + 8\right) + 8(2-\beta)(1-\beta)^2$$

It is easy to verify that $\overline{x} > \underline{x}$ for any $\beta \in (0,1)$.

We need one further assumption on the parameters of the model (for interiority).

$$\textbf{(A4)} \quad \begin{aligned} 9\gamma > I(\beta) \triangleq \; & \left(\tfrac{a}{c} - 1\right)(2-\beta) + (5\beta^2 - 12\beta + 8) + \\ & + \sqrt{\left(\left(\tfrac{a}{c} - 1\right)(2-\beta) + (5\beta^2 - 12\beta + 8)\right)^2 - 24\tfrac{a}{c}(2-\beta)(1-\beta)^2} \end{aligned}$$

We begin with a characterization of the set of PSNE in the R&D game with payoffs given in (4.1), and thus of subgame-perfect equilibria of the two-stage game.

[9] In contrast to our model, each player's payoff function in AW is instead globally submodular in (x_i, x_j) so that reaction curves have the same shape as those depicted in Figure 1a.

المنارة للاستشارات
www.manaraa.com

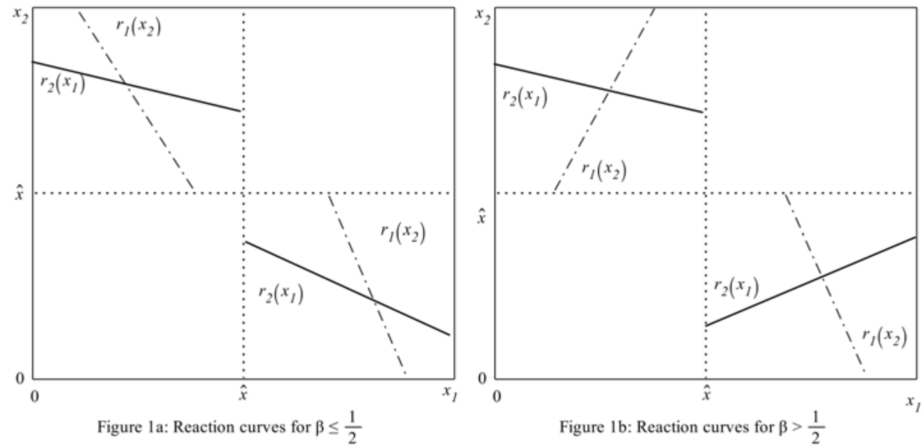

Figure 4.1: Reaction curves for different values of β

Proposition 4.2. *Under Assumptions (A1)-(A4), the R&D game admits a unique pair of PSNE of the form* $(\overline{x}, \underline{x})$ *and* $(\underline{x}, \overline{x})$.

Thus, although firms are ex-ante identical, only asymmetric equilibrium pairs of R&D investments prevail. This gives rise endogenously to a high R&D firm (called the innovator) and a low R&D firm (called the imitator).

As in the stochastic version of the model, the equilibrium levels of R&D investments are asymmetric due to the nonconcavity of the net profit function F along the 45° line. By Lemma 1, reaction curves jump downward over the diagonal at $\widehat{x}$ as indicated on Figure 1 so that, in equilibrium, a firm will always either spend less than its rival so as to free ride on the latter's R&D investment through spillovers, or spend more if the other firm's investment is too low in order to benefit from a competitive advantage over its rival in the product market. Notice that (A4) ensures that the two equilibrium pairs $(\overline{x}, \underline{x})$ and $(\underline{x}, \overline{x})$ are interior solutions. Instead, if (A1) through (A3) are satisfied, but (A4) is not, we have a boundary equilibrium of the

www.manaraa.com

form $\left(\overline{x}^B, \underline{x}^B\right)$ and $\left(\underline{x}^B, \overline{x}^B\right)$ where $\overline{x}^B = c$ and $\underline{x}^B = 4\frac{(a-2c(1-\beta))(1-\beta)}{9\gamma-8(\beta-1)^2}$. Figure 2 graphs assumptions (A2) through (A4) in the parameter space $(\beta, 9\gamma)$ and shows whether an interior or a boundary equilibrium prevails.

As long as $\beta > 0$, endogenous heterogeneity of firms will prevail with certainty in the present model. The one -dimensional nature of the R&D process gives rise naturally to one-way spillovers, which in turn provide incentives for firms to break off into an innovator and an imitator.[10] While a similar outcome prevails on average in the AW model, endogenous heterogeneity of firms materializes only with probability $(1-\beta)$, i.e., when no spillover takes place ex post.

Two special cases of the spillover parameter are worth highlighting. When $\beta = 1$, R&D is a pure public good, and the equilibrium autonomous and effective R&D levels, which reflect complete free-riding on the part of the follower (firm 2) as one would expect, are[11]

$$\overline{x} = \frac{4(a-c)}{9\gamma-2}, \ \underline{x} = 0 \ \text{ and } \ X_1 = \ X_2 = \frac{4(a-c)}{9\gamma-2}.$$

When $\beta = 0$, R&D is a pure private good, and the equilibrium autonomous and effective R&D levels reduce to

$$\overline{x} = \underline{x} = X_1 = \ X_2 = \frac{4(a-c)}{9\gamma-4}.$$

[10]As such, the present paper joins a recent trend of research in applied theoretical economics dealing with the endogenous emergence of asymmetric outcomes pertaining to ex ante identical agents. This is generally referred to as symmetry-breaking (a term borrowed from theoretical physics), or endogenous heterogeneity. See, inter alia, Matsuyama (2002), Amir, Garcia and Knauff (2010), Basu, Basu and Cordella (2016), Yazici (2016), Acemoglu et. al. (2017), and Chatterjee (2017).

[11]Indeed, conditional on being a follower, a firm has a dominant strategy of doing no R&D, as reflected by a reaction cruve identically equal to 0 in (4.2).

المنارة للاستشارات
www.manaraa.com

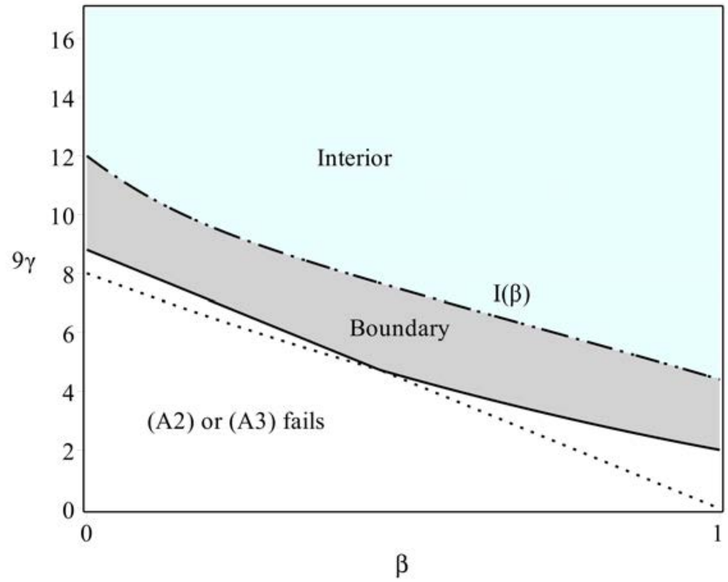

Figure 4.2: Type of equilibrium

4.4 R&D cooperation

In this section, we examine R&D cooperation by means of a joint lab, which allows firms to jointly appropriate the outcome of R&D investments in one and the same lab, while equally sharing the associated cost. This cooperation scenario was introduced in Amir (2000) both as a maximal R&D cooperation scenario and as a useful benchmark due to the absence of any spillover effects. This case will be referred to as Case J.

Recall that in models featuring the standard multi-directional spillover process with input spillovers (as in Spence, 1984 and Kamien, Muller and Zang, 1992), the so-called R&D cartel with spillover parameter internally increased to its maximal value of 1 is equivalent to a joint lab (as shown in Amir, 2000). This equivalence

www.manaraa.com

justifies viewing the joint lab as a maximal R&D cooperation scenario. In addition, this cooperation scenario delivers superior overall performance; it is shown in Kamien, Muller and Zang (1992) to dominate the other commonly used scenarios in terms of resulting firms' propensities for R&D, firms' profit and consumer surplus (and thus also social welfare).

In line with its superior performance, the interest in this form of cooperation in the present study will be seen to also lie in the fact that it coincides with a constrained version of the second-best outcome.

Under this scenario, the joint lab chooses a level of R&D that maximizes the sum of firms' profits, net of the (shared) R&D cost. Thus the problem of the joint lab is

$$\max_{x\in[0,c]} \{\frac{2}{9}(a-c+x)^2 - \frac{\gamma}{2}x^2\}.$$

The maximization yields the following per firm optimal level of investment

$$x_J = \begin{cases} 4\frac{a-c}{9\gamma-4} & \text{if } 9\gamma > 4\frac{a}{c} \\ c & \text{otherwise} \end{cases}$$

The next proposition provides a comparison of the joint lab's R&D investment with those of the noncooperative game.

Proposition 4.3. *Under Assumptions (A1)-(A4), the comparison of the equilibrium R&D levels in cases J and N is as follows:*

(i) $x_J = \overline{x} = \underline{x} = \frac{4(a-c)}{9\gamma-4}$ *if and only if* $\beta = 0$.

(ii) $x_J < \bar{x}$ *if* $9\gamma < 4(1-\beta)(4-3\beta)$ *and* $x_J > \bar{x}$ *otherwise.*

www.manaraa.com

(iii) $x_J > \underline{x} + \beta\left(\bar{x} - \underline{x}\right).$

(iv) *If both* $(\overline{x}, \underline{x})$ *and* x_J *are interior, then total effective cost reductions under the joint lab (case J) dominate those of the noncooperative setting (case N).*

The first part of the result captures the fact that, without any spillovers, the firms not only undertake the same level of R&D at equilibrium, but also that they actually undertake the same level as they would in a joint lab.

The second part says that the innovator invests more in R&D than the joint lab if the spillover parameter and/or the R&D costs are low enough. Intuitively, in the noncooperative setting, the prospect of efficiency gains when competing in the product market with a weaker rival boosts R&D investments in the first stage if these two conditions are satisfied. Conversely, its incentives to exert R&D effort are undermined if either the associated cost is large or the fraction of its cost reduction that spills over the imitator is high. Consequently, in this case, the joint lab reaches a higher level of cost reduction by both splitting the cost of undertaking R&D among firms and suppressing the free-rider issue.

Not surprinsingly, the level of R&D performed by the imitator in the noncooperative case is instead strictly lower than the joint lab's optimal cost reduction for any R&D cost and any spillover rate since R&D competition leaves scope for free riding over the innovator's investment due to the existence of spillovers.

Finally, for interior solutions, the total effective cost reduction achieved by means of cooperation via a joint lab is greater than in the noncooperative case. This last finding is in line with past results in the literature on the joint lab's superiority

المنارة للاستشارات
www.manaraa.com

in terms of the resulting propensity for R&D (Kamien et. al., 1992 and Amir, 2000).

Next, we examine the impact of R&D cooperation on firms' equilibrium profit. Equilibrium per-firm profit under a joint lab is given by

$$\tilde{F}(x_J) = \begin{cases} \frac{(a-c)^2\gamma}{9\gamma-4} & \text{if } 9\gamma > 4\frac{a}{c} \\ \frac{a^2}{9} - \frac{\gamma c^2}{4} & \text{otherwise} \end{cases}$$

Even though firms inherit the same cost structure under a joint lab, thereby dissipating firms' total profit in the product market, cooperation through a joint lab allows firms to share R&D costs thereby avoiding the inefficiencies associated with the free-rider issue inherent to the noncooperative setting. The next result establishes that the latter effect dominates the former, thereby making the industry strictly better off when cooperating through a joint lab.

Proposition 4.4. $2\tilde{F}(c) \geq F(\overline{x}, \underline{x}) + F(\underline{x}, \overline{x})$ *and* $2\tilde{F}(x_J) \geq F\left(\overline{x}^B, \underline{x}^B\right) + F\left(\underline{x}^B, \overline{x}^B\right)$.

As a brief conclusion for this section, it may be said that, while the endogenous asymmetry can reverse some of the established conclusions on the superiority of the joint lab in the literature, these conclusions can be restored when considering aggregate performance.

4.5 Welfare analysis

In this section, we consider a benevolent central planner with a second-best mandate, i.e., one that is endowed with the authority to decide on R&D investments but has no control or influence over the firms' market conduct, once R&D levels have been selected. We first examine the case where the planner is constrained to

المنارة للاستشارات
www.manaraa.com

impose symmetric R&D expenditures across firms (i.e, to satisfy the principle of equal treatment of equals). We shall refer to this planner's scenario as Case P_S (for second-best planning under symmetric treatment of firms).[12]

We then consider the case where the social planner is unconstrained in its choices of R&D levels and can thus exploit the benefits of asymmetric choices. We abbreviate this planner's scenario as Case P_A (for second-best planning under possible asymmetry). Finally, we compare the two different planning solutions and examine the social costs of imposing equal treatment among firms.

Assume w.l.o.g. that $x_1 > x_2$, and define social surplus (welfare) in the usual way as the sum of firms' profit and consumer surplus, i.e., we have

$$\begin{aligned} S(x_1, x_2, q_1, q_2) = & (q_1 + q_2)\left(a - \frac{(q_1 + q_2)}{2}\right) - (c - x_1)q_1 \\ & - (c - (x_2 + \beta(x_1 - x_2)))q_2 - \frac{\gamma}{2}\left(x_1^2 + x_2^2\right) \end{aligned}$$

Given the Cournot equilibrium in the second stage of the game, one can write social welfare as a function of the R&D (first-period) decisions by substituting the Cournot outputs into $S(x_1, x_2, q_1, q_2)$. This yields

$$\begin{aligned} W(x_1, x_2) = & \frac{1}{18}\left[8(a-c)\left((a-c) + x_1(1+\beta) + x_2(1-\beta)\right)\right. && (4.3) \\ & - x_1^2\left(9\gamma + 14\beta - 11\beta^2 - 11\right) - x_2^2\left(9\gamma - 11(1-\beta)^2\right) \\ & \left. + 2x_1x_2(11\beta - 7)(1-\beta)\right] && (4.4) \end{aligned}$$

[12]It is worth stressing that this principle is widely taken for granted in the formulation of public policy. As such, it is generally not even a subject of debate, although, as we shall see, this is not necessarily in society's interest in the present context.

www.manaraa.com

4.5.1 Symmetric second-best planner's solution

In this subsection, the planner is constrained to select symmetric R&D expenditures for the two identical firms, and thus to satisfy the principle of equal treatment of equals when engaging in any sort of regulation of firms. The main finding here is that the planner's solution yields the same R&D for each firm as a joint lab.

The problem of the central planner is thus (see (4.3))

$$\max_{(x_1,x_2)\in[0,c]^2}\{W(x_1,x_2) : x_1 = x_2\}$$

Upon a simple computation, the optimal symmetric per firm investment level and the corresponding symmetric-optimal second-best welfare are given by

$$x_s = \frac{4\,(a-c)}{9\gamma-4} \text{ and } W\,(x_s,x_s) = \frac{4\gamma\,(a-c)^2}{9\gamma-4}.$$

As seen by simple inspection, the symmetry-constrained socially optimal level of R&D coincides with the optimal R&D level of the joint lab, i.e., $x_s = x_J$, and therefore

$$x_s = \overline{x} = \underline{x} = \frac{4(a-c)}{9\gamma-4} \text{ if and only if } \beta = 0$$

This coincidence of R&D levels means that the joint lab is a socially optimal form of R&D cooperation under the equal treatment restriction. Furthermore, non-cooperative R&D yields a second-best symmetry-constrained socially optimal level of R&D as long as R&D spillovers are fully absent. The latter requirement is quite unrealistic, since spillovers are typically considered as an unavoidable characteristic of the technological environment in an industry.[13]

[13]Naturally, the level of spillovers may be influenced by location patterns, patent policy,

www.manaraa.com

Another direct implication of this outcome is that a joint lab emerges as one possible and practical way to implement a symmetric socially optimal scenario without involving a central planner at all.

Furthermore, since the joint lab avoids the duplication of R&D costs by definition, it leads to a welfare level W_J that is clearly strictly higher then the symmetric-optimal second-best welfare, i.e., $W_J > W(x_s, x_s)$.[14] A brief calculation shows that

$$W_J = \frac{4(9\gamma - 2)(a-c)^2 \gamma}{(9\gamma - 4)^2},$$

The fact that $x_s = x_J$ yields the following from the results on the joint lab.

Proposition 4.5. *The comparison of the equilibrium R&D levels in cases P_S and N is as follows:*

(i) $x_s = x_J = \overline{x} = \underline{x} = \frac{4(a-c)}{9\gamma - 4}$ *if and only if* $\beta = 0$.

(ii) $x_s < \bar{x}$ *if* $9\gamma < 4(1-\beta)(4-3\beta)$ *and* $x_s > \bar{x}$ *otherwise.*

(iii) $x_s > \underline{x}$ *always.*

Again, the case of no spillovers yields an exceptional outcome worth highlighting. Not only does the non-cooperative solution coincide with the joint lab, it also yields a symmetry-constrained second best socially optimal levels of R&D. The direct implication of this simple observation is obvious yet quite important: With no spillovers, the market solution is second-best efficient (albeit in a constrained man-

---

and other factors. Nevertheless, it is unrealistic to assume that they may be driven down all the way to zero.

[14] More precisely, $W_J = W(x_s, x_s) - \frac{\gamma x_s^2}{4}$. Note that W_J cannot be expressed via the function $W(\cdot, \cdot)$.

المنارة للاستشارات
www.manaraa.com

ner), so laissez-faire, as opposed to both intervention or a joint lab, is the way to go.

While the unconstrained social planner inherits the incentive to create a high R&D firm and a low R&D firm, just like firms in Case N, the planner would generate a smaller spread between the firms when R&D costs are low (or γ small), and a higher level of R&D for both firms when R&D costs are high. The latter outcome is of course the standard implication of social planning for R&D, the aim being to correct for the market's tendency to supply too little R&D, due to the well-known and documented gap between private and social returns to R&D (see e.g., Griliches, 1995 and Bernstein and Nadiri, 1988). On the other hand, it is certainly noteworthy that, with low R&D costs, the market outcome leads to more R&D for the innovator than the socially optimal solution.

Whether symmetric-optimal welfare $W(x_s, x_s)$ is superior to that of the non-cooperative setting instead depends on the magnitude of R&D costs. Two conflicting effects need to be considered. On the one hand, imposing symmetric R&D investments implies that firms face the same unit cost when competing in the product market, which in turn leads to total profit dissipation. On the other hand, since $x_s = x_J$, total effective cost reductions are higher in the symmetric planner's solution (see Proposition 4.2). It follows that consumers benefit from a lower price under the latter solution. The following result characterizes regions of parameters for which either effect dominates.

Proposition 4.6. *The ranking of welfare levels under Cases N, J and P_S is as*

www.manaraa.com

follows:

$$W_J \geq \begin{cases} W(\bar{x}, \underline{x}) > W(x_s, x_s) & if\, 9\gamma < K_4 \\ \\ W(x_s, x_s) > W(\bar{x}, \underline{x}) & otherwise \end{cases}$$

where

$$K_4 = \frac{1}{2}\left(43\beta^2 - 102\beta + 55\right) + \frac{1}{2}\sqrt{1057 - 4212\beta + 5870\beta^2 - 3396\beta^3 + 697\beta^4}$$

Not surprisingly, total welfare when firms cooperate through a joint lab exceeds that of the noncooperative setting. Indeed, both the innovator and the imitator get a strictly higher profit when cooperating (cf. Section 4.4). Moreover, since the second-best welfare maximizing symmetric R&D investments coincide with those of the joint lab, Proposition 4.2 applies. Namely, total effective cost reductions are higher than those achieved in the noncooperative setting so that aggregate production costs are lower whenever a central planner intervenes in the R&D game. It directly follows that firms charge a lower price, and that consumer surplus is higher, i.e., $CS(x_s, x_s) \geq CS(\bar{x}, \underline{x})$.

Recall that Kamien et. al (1992) demonstrated that, in the analogous model but with two-way spillovers, the joint lab (or the cartelized R&D joint venture as they described it in an equivalent manner) is superior in terms of propensity for R&D and social welfare to the other three scenarios examined in that paper. Here, we show that, with one-way spillovers, the joint lab actually yields the socially optimal level of R&D subject to the equal treatment restriction.

The key conclusion of this part is that a joint lab may be regarded as a simple and non-interventionist manner of actually implementing a second-best socially

المنارة للاستشارات
www.manaraa.com

optimal outcome for a duopoly with one-way spillovers. Indeed, while second-best planning is often taken as a useful benchmark for policy analysis, a joint lab represents an actual institution that can not only lead to the social levels of R&D, but also avoid the duplication costs in carrying out the R&D.

Next, we relax the assumption that the social planner imposes equal treatment across firms.

4.5.2 Asymmetric second-best planner's solution

In this part, the problem of the social planner is now to choose a pair of (possibly asymmetric) R&D investments that maximizes total welfare, as given by Eq. (3), that is

$$(\overline{x}_a, \underline{x}_a) \in \underset{(x_1,x_2)\in[0,c]^2}{\operatorname{argmax}} W(x_1, x_2) \tag{4.5}$$

Intuitively, one would expect the global argmax of social welfare to be asymmetric, as a result of the well-known fact that Cournot equilibrium industry profit is convex in firms unit costs. In other words, industry profit tends to be higher when firms are asymmetric in terms of unit costs, and this property is inherited by social welfare (see e.g., Salant and Shaffer, 1998, 1999, and Soubeyran and Van Long, 1999).

Indeed, this intuition is confirmed by the solution, as it may be easily verified that the optimal investment levels are given by

$$\overline{x}_a = \frac{4}{D_a}[\gamma(\beta+1) - 2(1-\beta)^2](a-c) \text{ and } \underline{x}_a = \frac{4}{D_a}[\gamma - 2(1-\beta)](1-\beta)(a-c),$$

where

$$D_a \triangleq 9\gamma^2 - 2\gamma\left(11\beta^2 - 18\beta + 11\right) + 8(1-\beta)^2$$

المنارة للاستشارات
www.manaraa.com

It is easy to check that this solution is always asymmetric for any non-zero value of β, i.e. that

$$\overline{x}_a \geq \underline{x}_a \text{ with equality if and only if } \beta = 0.$$

Therefore, in industries with non-zero one-way spillovers, the social planner always faces a clear incentive for unequal treatment of regulated firms, even when these are ex ante symmetrical.[15]

Two special cases of the spillover parameter are worth reporting. When $\beta = 1$, R&D is a pure public good, and the second-best autonomous and effective R&D levels, reflecting the fact that the planner takes full advantage of the perfect spillovers for the follower (firm 2), are

$$(\overline{x}_a, \underline{x}_a) = (\frac{8(a-c)}{9\gamma - 8}, 0) \text{ and } X_1 = X_2 = \frac{8(a-c)}{9\gamma - 2}.$$

When $\beta = 0$, R&D is a pure private good, and the autonomous and effective R&D levels reduce to

$$\overline{x} = \underline{x} = X_1 = X_2 = \frac{4(a-c)}{9\gamma - 4}.$$

The corresponding optimal level of social welfare for any $\beta \in [0, 1]$ is

$$W(\overline{x}_a, \underline{x}_a) = \frac{4}{D_a}\gamma[\gamma - 2(1-\beta)^2](a-c)^2.$$

[15]A lengthy computation shows that $0 < x_2^W < x_1^W < c$ if $9\gamma > 18(1-\beta)$ and $9\gamma > Z_1$ where

$$\begin{aligned} Z_1 &= 2\frac{a}{c}(1+\beta) - (11\beta - 9)(1-\beta) + \\ &+\frac{1}{c}\sqrt{(11\beta-9)^2(\beta-1)^2 + 4(\frac{a}{c})^2(\beta+1)^2 + 4\frac{a}{c}(\beta-1)(11\beta^2 - 16\beta + 9)}. \end{aligned}$$

المنارة للاستشارات

www.manaraa.com

Since symmetric choices of R&D levels are one option that the social planner has in the optimization problem (4.5), it follows that $W(\overline{x}_a, \underline{x}_W) > W(x_W, x_W)$, as is easily verified by direct calculation. Nevertheless, despite its sub-optimality, the constrained-symmetric solution may well be of substantial real-life interest, since implementing an asymmetric solution on a priori identical firms is likely to be politically infeasible. It would be akin to forging a national champion and a weak firm out of two equally efficient firms.

Our next result compares the second-best welfare maximizing asymmetric R&D investments with those of the noncooperative setting, as well as the associated effective cost reductions.

Proposition 4.7. *The second-best welfare maximizing asymmetric R&D investments satisfy:*

(i) $\overline{x}_a > \bar{x}$ *and* $\underline{x}_a > \underline{x}$ *if* $\beta > \frac{2}{3}$, *while* $\underline{x}_a < \underline{x}$ *if both* $\beta \leq \frac{2}{3}$ *and* $9\gamma > \frac{2(1-\beta)(23\beta-11-11\beta^2)}{(3\beta-2)}$

(ii) $(1+\beta)\overline{x}_a + (1-\beta)\underline{x}_a > (1+\beta)\bar{x} + (1-\beta)\underline{x}$

For Part (i), it is noteworthy that for small spillover rates, the social planner would actually dictate a lower R&D level for the imitator. The intuition for this finding is that the social planner is more apt than the non-cooperative solution to take advantage of the aforementioned asymmetry premium for social welfare, and thus more prone to a higher dispersion in R&D levels.

Part (ii) of this result is not surprising, since it simply confirms for the par-

المنارة للاستشارات
www.manaraa.com

ticular setting at hand a well-known general fact about innovation in general: That the market typically undersupplies R&D, due to well-established market failures, in particular to the imperfectly appropriable nature of process R&D here. Thus even a second-best social planner would typically choose to generate higher levels of effective R&D.

Furthermore, while total welfare achieved under symmetric regulation is inferior to that induced by the joint lab, the next result states on the contrary that asymmetric regulation often welfare-dominates the joint lab.

Proposition 4.8. *Total welfare induced by the asymmetric second-best welfare maximizing R&D investments satisfies the following:*

(i) $W(\overline{x}_a, \underline{x}_a) > W(x_s, x_s)$

(ii) $W(\overline{x}_a, \underline{x}_a) > W_J$ *if either* $\beta \geq \frac{\sqrt{2}}{2}$ *or* $\beta < \frac{\sqrt{2}}{2}$ *and* $9\gamma < Z_3$ *where*

$$Z_3 = (1-\beta)\,\frac{7\beta - 11 - \sqrt{193\beta^2 - 154\beta + 49}}{2\beta^2 - 1}$$

Part (i) is an obvious statement in that it captures the premium to the social planner of having fully flexible choices in firms' R&D levels. A quantitative assessment of the welfare loss to being subject to the symmetry constraint in R&D choices is investigated in the next subsection.

An intuitive understanding for part (ii) of this result may be achieved as follows. The comparison at hand involves two issues with respect to which the two scenarios hold opposite positions: R&D duplication costs and symmetry of R&D choices. The joint lab has the advantage of avoiding R&D duplication costs but forces

المنارة للاستشارات
www.manaraa.com

firms to settle for symmetric R&D levels. On the other hand, unconstrained welfare maximization faces R&D duplication costs but allows asymmetric R&D choices. Part (ii) states that the second best asymmetric regulation (or Case P_A) welfare-dominates the joint lab (Case J) when either the spillover parameter is high enough or else when R&D is relatively less costly.

Therefore, overall the main message of this Proposition is that the flexibility to choose asymmetric R&D levels often contributes substantially to social welfare. We now examine this issue in a quantitative sense.

4.5.3 The welfare cost of equal treatment

We have seen that, in industries with (non-zero) one-way spillovers, the social planner always has an incentive to engage in discriminatory regulation of the two firms in order to maximize social welfare. However, in most societies, political, moral and other fairness considerations will dictate that the social planner engage instead in equal treatment of regulated firms, in total disregard of any resulting loss of social welfare. In this subsection, we investigate the value of the welfare loss due to the equal treatment constraint, and its comparative statics as the parameters of the model vary exogenously.

The welfare loss is defined as the difference between asymmetric and symmetric optimal second best welfare, i.e.,

$$L = W(\overline{x}_a, \underline{x}_a) - W\left(x_s, x_s\right).$$

Using the expressions for the two welfare levels $W(\overline{x}_a, \underline{x}_a)$ and $W\left(x_s, x_s\right) \triangleq W$ given

www.manaraa.com

above, one arrives upon simplification at

$$L = \frac{16\beta^2\gamma^2 (a-c)^2}{(9\gamma-4) D_a}.$$

It is easy to verify that $\frac{\partial L}{\partial \beta} > 0$. This is intuitive, since it simply reflects that, being due to the nature of the spillover process, the scope for endogenous heterogeneity of firms' post-R&D costs increases with the size of spillovers.

Furthermore, as β increases from 0 to 1, it may be verified that the welfare loss L increases from 0 to $L = \frac{16\gamma^2(a-c)^2}{(9\gamma-4)(9\gamma^2-8\gamma+8)}$.

Therefore, since W is independent of β, $\frac{L}{W}$ can be as high as

$$\frac{16\gamma^2 (a-c)^2}{(9\gamma-4)(9\gamma^2-8\gamma+8)} / \frac{4\gamma (a-c)^2}{(9\gamma-4)} = \frac{4\gamma}{9\gamma^2-8\gamma+8}$$

Maximizing the latter expression with respect to γ yields a unique argmax of $\gamma^* \approx 0.943$, and a corresponding maximal value of $\frac{L}{W}$ equal to 0.446.[16]

We have just established part (ii) of the following result (part (i) follows directly from evaluating and signing $dL/d\beta$ and $dL/d\gamma$. This is easy to do, thus left to the reader).

Proposition 4.9. *The welfare loss L due to equal treatment in R&D regulation satisfies:*

(i) L is increasing in β and in $(a-c)$, and decreasing in γ.

(ii) The maximal welfare cost of equal treatment in relative terms, $\frac{L}{W}$, is 44.6%.

[16] The ratio $\frac{L}{W}$ turns out (evaluated at $\beta = 1$) to be strictly quasi-concave in γ, so that the first-order condition is necessary and sufficient for a unique argmax. In addition, Assumptions (A2)-(A3) are easily seen to be satisfied around $\gamma^* \approx 0.943$.

المنارة للاستشارات

www.manaraa.com

This number is remarkably high, even when understood as just a conceivable upper bound on the relative size of welfare loss. Indeed, the actual loss for a particular industry will depend on the specific values of β and γ (the lower bound of this loss is clearly 0, which is easily seen to be achieved for a spillover value of $\beta = 0$, due to the symmetric solution then). This illustration clearly indicates that this ubiquitous aversion to unequal treatment can lead to quantitatively significant losses.

The dichotomy between normative and positive (or politically-constrained) efficiency emerges in several different settings in the process of implementing various aspects of public policy. Different manifestations of the same fundamental issue may be seen in a number of different studies covering various areas of economics, including for instance Spencer and Brander (1985), Salant and Shaffer (1992, 1999), Matsuyama (2002), Basu, Basu and Cordella (2016), Yazici (2016), Acemoglu et. al. (2017), and Chatterjee (2017), among many others.

4.6 Conclusion

This paper has investigated the properties of a symmetric two-period R&D model that departs from the standard setting by adopting a deterministic one-way spillover structure. The latter is a reflexion of the one-dimensional nature of the R&D process. Though firms are ex-ante identical, one obtains a unique pair of asymmetric equilibria in terms of R&D investments. Thus the roles of R&D innovator and imitator are endogenously determined as a direct consequence of the one-way spillover structure. This establishes a simple link between the nature of the R&D process

www.manaraa.com

in an industry –including the associated spillover –and the emergence of inter-firm heterogeneity in that industry.

The main part of the paper provides a welfare analysis in which we examine the usual question of how distortive the non-cooperative equilibrium is, in terms of propensity for R&D and equilibrium welfare. To this end, we consider a realistic second-best social planner who selects firms' R&D levels but does not control their Cournot market conduct. We also compare the performance of the joint lab as an R&D cooperation scenario with the second best optimum. Under the constraint of symmetric treatment of the firms by the planner, the socially optimal solution yields the same R&D level as the joint lab. It follows that the latter is a practical way to realize the second best level of R&D without direct intervention.

Finally, due to the fact that the same forces that lead to asymmetric Nash equilibrium in R&D levels also lead to asymmetric (unconstrained) social optima, we investigate in some detail the social costs (or welfare loss) of imposing the politically-motivated constraint of symmetric R&D investments among firms. We find that this social cost can reach the highly significant level of 46% in relative terms.

4.7 Proofs

Proof of Lemma 1

The reaction function r as given by Eq. (4.2) is not continuous since, letting $x^{S1} = r_1(x^{S1})$ for $x_1 \geq x_2$ and $x^{S2} = r_1(x^{S2})$ for $x_1 \leq x_2$, one obtains

$$x^{S1} = \frac{2\,(a-c)\,(2-\beta)}{(9\gamma - 2\,(2-\beta))}, \text{ and } x^{S2} = \frac{4\,(a-c)\,(1-\beta)}{(9\gamma - 4\,(1-\beta))}$$

www.manaraa.com

with $x^{S1} > x^{S2}$. Hence, the reaction function has a downward jump, and letting $\widehat{x}$ be the solution to $U(r_1(\widehat{x}), \widehat{x}) = L(r_1(\widehat{x}), \widehat{x})$, we have that

$$\widehat{x} = \frac{(a-c)\left(\sqrt{1+\frac{2\beta(4-3\beta)}{\left(9\gamma-2(\beta-2)^2\right)}}-1\right)}{\left(2\beta-1+(1-\beta)\sqrt{1+\frac{2\beta(4-3\beta)}{\left(9\gamma-2(\beta-2)^2\right)}}\right)} \tag{4.6}$$

Furthermore, $\hat{x}$ is unique since both U and L are monotonic in x_2. U is decreasing in x_2 for all $\beta \in [0,1]$, while L either increases with x_2 for $\beta > 1/2$ or decreases with x_2 slower than U. □

Proof of Proposition 1

A lengthy but simple computation establishes that $\bar{x}, \underline{x}$ as given in the text satisfy $\overline{x} > \widehat{x}$ if $9\gamma > I_1$ and $\underline{x} < \widehat{x}$ if $9\gamma > I_2$, with $\hat{x}$ as defined by Eq. (5) and

$$\begin{aligned} I_1 &= \left(5\beta^2-12\beta+8\right)+\sqrt{13\beta^4-48\beta^3+68\beta^2-48\beta+16}, \\ I_2 &= \left(5\beta^2-12\beta+8\right)+\sqrt{73\beta^4+224\beta^2-216\beta^3-96\beta+16}, \end{aligned}$$

Straightforward computations then establish that $I(\beta) > I_1$ and $I(\beta) > I_2$. Hence, if assumptions (A1) through (A4) hold, the pair of PSNE $(\overline{x}, \underline{x})$ and $(\underline{x}, \overline{x})$ is unique. □

Proof of Proposition 2

(i) This is seen by inspection.

(ii) We have that

$$x_J - \bar{x} = \frac{4(a-c)}{9\gamma-4} - \frac{2\,(a-c)\,(2-\beta)\left(3\gamma-4\,(\beta-1)^2\right)}{D_N}$$

Simplifying and rearranging then leads to

$$sign(x_J - \bar{x}) = sign\left(28\beta + 9\gamma - 12\beta^2 - 16\right) < 0$$

المنارة للاستشارات
www.manaraa.com

if and only if $9\gamma < 4(1-\beta)(4-3\beta)$.

(iii) Similarly, it can be shown that

$$\begin{aligned} x_J - (\underline{x} + \beta(\bar{x} - \underline{x})) &= \frac{4(a-c)}{9\gamma - 4} \\ &\quad - \frac{2(a-c)\left(3(\beta^2 - 2\beta + 2)\gamma - 4(2-\beta)(\beta-1)^2\right)}{27\gamma^2 - 6\gamma(5\beta^2 - 12\beta + 8) + 8(2-\beta)(1-\beta)^2} \\ &> 0 \text{ if } 9\gamma > \frac{4(1-\beta)(5-3\beta)}{(2-\beta)}. \end{aligned}$$

which can be shown to hold for all the parameter values for which the Nash equilibrium is interior, or if $I(\beta) > \frac{4(1-\beta)(5-3\beta)}{(2-\beta)}$.

(iv) Total cost reductions achieved under cooperation through a joint lab formation dominate those of the noncooperative regime if

$$\begin{aligned} \frac{8(a-c)}{9\gamma-4} > &(1+\beta)\frac{2(a-c)(2-\beta)\left(3\gamma - 4(\beta-1)^2\right)}{D_N} \\ &+ (1-\beta)\frac{4(a-c)(1-\beta)(3\gamma - 2(\beta-1)(\beta-2))}{D_N} \end{aligned}$$

$$\Leftrightarrow 9\gamma > \frac{12(1-\beta)(3-2\beta)}{(3-\beta)}$$

which holds at an interior equilibrium since $I(\beta) > \frac{12(1-\beta)(3-2\beta)}{(3-\beta)}$. □

Proof of Proposition 3

We first check that $2\tilde{F}(x_J) > F(\overline{x}_B, \underline{x}_B) + F(\underline{x}_B, \overline{x}_B)$ holds for $4\frac{a}{c} < 9\gamma \leq I(\beta)$. It may be verified that the difference $2\tilde{F}(x_J) - F(\overline{x}_B, \underline{x}_B) - F(\underline{x}_B, \overline{x}_B)$ is positive if

المنارة للاستشارات
www.manaraa.com

and only if

$$-729c^2\gamma^4 + 162c\gamma^3\left(13c\beta^2 + (2a-26c)\beta + 2a + 13c\right)$$

$$-72\left(\left(21-32\beta+16\beta^2\right)(1-\beta)^2c^2 - 2a(1-\beta)\left(-6+6\beta+\beta^2\right)c + 2a^2\right)\gamma^2$$

$$+32(1-\beta)^2\left(8(1-\beta)^2c^2\right.$$

$$\left.+2a(7-8\beta)(1-\beta)c + a^2\left(9-2\beta+\beta^2\right)\gamma - 128a^2(1-\beta)^4 < 0\right.$$

Numerical computations then establish that this inequality holds for $9\gamma \in [4\frac{a}{c}, I(\beta)]$.

In a similar fashion, we now shall show that $2\tilde{F}(c) > F(\overline{x}, \underline{x}) + F(\underline{x}, \overline{x})$ for $I(\beta) < 9\gamma < 4\frac{a}{c}$. A lengthy computation establishes that the sign of the difference $2\tilde{F}(c) - F(\overline{x}, \underline{x}) - F(\underline{x}, \overline{x})$ is the same as that of L_6, where

$$L_6 = -6561c^2\gamma^5 + \gamma^4\left(14\,580c^2\beta^2 - 34\,992c^2\beta + 20\,412c^2 + 5832ac\right)$$

$$+\gamma^3(972a^2\beta^2 - 1944a^2\beta - 14\,904ac\beta^2 + 34\,992ac\beta c^2$$

$$-20\,736ac - 8100c^2\beta^4 + 42\,768c^2\beta^3 - 80\,676c^2\beta^2 + 64\,152c^2\beta - 18\,144)$$

$$+\gamma^2(9072a^2\beta^3 - 2232a^2\beta^4 - 11\,592a^2\beta^2$$

$$+4320a^2\beta + 576a^2 + 11\,664ac\beta^4 - 56\,160ac\beta^3 + 101\,520ac\beta^2$$

$$-81\,216ac\beta + 24\,192ac - 4320c^2\beta^5 + 21\,816c^2\beta^4 - 41\,904c^2\beta^3$$

$$+37\,368c^2\beta^2 - 14\,688c^2\beta + 1728c^2)$$

$$+\gamma(1152a^2\beta^6 - 7296a^2\beta^5 + 17\,664a^2\beta^4 - 19\,584a^2\beta^3$$

$$+8064a^2\beta^2 + 1536a^2\beta - 1536a^2 - 2304ac\beta^6$$

المنارة للاستشارات
www.manaraa.com

$$+18\,432ac\beta^5 - 59\,904ac\beta^4 + 101\,376ac\beta^3 - 94\,464ac\beta^2$$

$$+46\,080ac\beta - 9216ac + 576c^2\beta^6 - 4608c^2\beta^5$$

$$+14\,976c^2\beta^4 - 25\,344c^2\beta^3 + 23\,616c^2\beta^2 - 11\,520c^2\beta + 2304 + c^2)$$

$$+\left(256a^2\beta^6 - 2048a^2\beta^5 + 6656a^2\beta^4 - 11\,264a^2\beta^3\right.$$

$$+10\,496a^2\beta^2 - 5120a^2\beta + 1024a^2$$

For apparent reasons, we relied on numerical computations then to demonstrate that $L_6 > 0$ for $9\gamma \in [I(\beta), 4\frac{a}{c}]$. □

Proof of Proposition 5

We first shall show that $W_J > W(x_s, x_s)$. We have that

$$4\frac{(9\gamma - 2)(a-c)^2\gamma}{(9\gamma - 4)^2} > 4\frac{(a-c)^2\gamma}{(9\gamma - 4)} \Leftrightarrow 9\gamma > 3$$

which holds from assumption (A2) and the fact that $9\gamma > 4\frac{a}{c}$.

Next, we establish that $W_J \geq W(\bar{x}, \underline{x})$. From section 4, we have that $\tilde{F}(x_J) > F(\underline{x}, \bar{x})$ and $\tilde{F}(x_J) > F(\bar{x}, \underline{x})$ for $9\gamma > \max\{4\frac{a}{c}, I(\beta)\}$. Hence, it directly follows that the industry's profit under the joint lab formation exceeds that of the noncooperative setting, i.e.

$$2\tilde{F}(x_J) > F(\underline{x}, \bar{x}) + F(\bar{x}, \underline{x}) \tag{4.7}$$

Therefore, a sufficient condition for $W_J \geq W(\bar{x}, \underline{x})$ to hold is that consumer surplus when firms cooperate through a joint lab is higher. The difference $CS(x_J, x_J) - CS(\bar{x}, \underline{x})$ is given by

$$2\left(\frac{3\gamma(a-c)}{(9\gamma-4)}\right)^2 - \frac{18\left(3\gamma + (1-\beta)(3\beta-4)\right)^2 (a-c)^2\gamma^2}{\left(27\gamma^2 - 6\gamma\left(5\beta^2 - 12\beta + 8\right) - 8(\beta-2)(\beta-1)^2\right)^2}$$

المنارة للاستشارات

www.manaraa.com

Straightforward computations then establish that $CS\left(x_J, x_J\right) - CS(\bar{x}, \underline{x}) \geq 0$ if and only if $K_6.K_7 \geq 0$, where

$$\begin{aligned} K_6 &= \left(4\left(1-\beta\right)\left(2\beta^2-9\beta+8\right)+54\gamma^2-3\gamma\left(19\beta^2-45\beta+32\right)\right) \\ K_7 &= \left(3\gamma\left(3-\beta\right)-4\left(1-\beta\right)\left(3-2\beta\right)\right). \end{aligned}$$

Both K_6 and K_7 are positive for all $9\gamma > I(\beta)$. Thus, we have that

$$CS\left(x_J, x_J\right) \geq CS(\bar{x}, \underline{x}) \tag{4.8}$$

Hence, (11) together with (12) establish the superiority of the joint lab in terms of welfare.

Finally, the difference $W(\bar{x}, \underline{x}) - W\left(x_s, x_s\right)$ is given by

$$\frac{\begin{array}{c} 2\gamma\left(a-c\right)^2\left[162\gamma^3 - -9\gamma^2\left(41\beta^2-96\beta+56\right)\right. \\ \left. +3\gamma\left(81\beta^2-224\beta+160\right)\left(1-\beta\right)^2 - 32\left(2-\beta\right)^2\left(1-\beta\right)^4\right] \end{array}}{\left(27\gamma^2-6\gamma(5\beta^2-12\beta+8)-8(\beta-2)(\beta-1)^2\right)^2} - 4\frac{(9\gamma-2)(a-c)^2\gamma}{(9\gamma-4)^2}$$

Simplifying and rearranging, we have that

$$W(\bar{x}, \underline{x}) > W\left(x_s, x_s\right) \Leftrightarrow 9\gamma \in \left(K_5, K_4\right)$$

with K_4 as indicated in the proposition and

$$K_5 = \frac{1}{2}\left(43\beta^2-102\beta+55\right) - \frac{1}{2}\sqrt{1057-4212\beta+5870\beta^2-3396\beta^3+697\beta^4}$$

where $K_5 < I(\beta)$. $\square$

المنارة للاستشارات
www.manaraa.com

Proof of Proposition 6

(i) Upon simplification, the sign of the difference $\overline{x}_a - \overline{x}$ is the same as that of $8\left(16 - 11\beta\right)\left(1-\beta\right)^3 + 81\gamma^2 - 18\gamma\left(1-\beta\right)\left(11 - 9\beta\right)$, which is strictly positive for $9\gamma > 18(1+\beta)$.

Likewise, it may be easily verified that the sign of $\underline{x}_a - \underline{x}$ is the same as that of $9\gamma\left(3\beta - 2\right) - 2\left(1-\beta\right)\left(23\beta - 11 - 11\beta^2\right)$. This expression is strictly positive if both $\beta > \frac{2}{3}$ and $9\gamma > 18(1+\beta)$, so that $\underline{x}_a > \underline{x}$. Instead, if either $\beta = \frac{2}{3}$, or $\beta < \frac{2}{3}$ and $9\gamma > \frac{2(1-\beta)\left(23\beta-11-11\beta^2\right)}{(3\beta-2)}$, then $\underline{x}_a < \underline{x}$.

(ii) As for total effective cost reductions, straightforward computations establish that $(1+\beta)\overline{x} + (1-\beta)\underline{x} - (1+\beta)\overline{x}_a - (1-\beta)\underline{x}_a < 0$ if $-9\left(1+\beta\right)\gamma^2 - 2\left(1-\beta\right)\left(-15 + 8\beta + 3\beta^2\right)\gamma + 8\left(2\beta - 3\right)\left(1-\beta\right)^3 < 0$, which holds for any $\beta \in [0,1]$ and $9\gamma > Z_1$. $\square$

Proof of Proposition 7

(i) We have that

$$\begin{aligned} W(\overline{x}_a, \underline{x}_a) - W\left(x_s, x_s\right) =& \frac{4\left(\gamma - 2\left(1-\beta\right)^2\right)\left(a-c\right)^2\gamma}{9\gamma^2 - 2\gamma\left(11\beta^2 - 18\beta + 11\right) + 8\left(1-\beta\right)^2} - 4\frac{\left(a-c\right)^2\gamma}{\left(9\gamma - 4\right)} \\ =& \frac{16\left(a-c\right)^2\beta^2\gamma^2}{\left(9\gamma-4\right)\left(9\gamma^2 - 2\gamma\left(11\beta^2 - 18\beta + 11\right) + 8\left(1-\beta\right)^2\right)} \\ >& 0 \end{aligned}$$

(ii) The difference $W(\overline{x}_a, \underline{x}_a) - W_J$ is given by

$$\frac{4\left(\gamma - 2\left(1-\beta\right)^2\right)\left(a-c\right)^2\gamma}{9\gamma^2 - 2\gamma\left(11\beta^2 - 18\beta + 11\right) + 8\left(1-\beta\right)^2} - 4\frac{\left(9\gamma - 2\right)\left(a-c\right)^2\gamma}{\left(9\gamma-4\right)^2}$$

المنارة للاستشارات
www.manaraa.com

so that $W(\overline{x}_a, \underline{x}_a) - W_J > 0$ if $16\,(1-\beta)^2 + 18\gamma^2\,(1-2\beta^2) - 4\gamma\,(1-\beta)\,(11-7\beta) > 0$, which holds if either $\beta < \frac{1}{2}\sqrt{2}$ and $9\gamma < Z_2$, or $\beta \geq \frac{1}{2}\sqrt{2}$ provided that $9\gamma > 18(1+\beta)$. $\square$

المنارة للاستشارات

www.manaraa.com

REFERENCES

[1] Acemoglu, D., J. Robinson and T. Verdier (2017). Asymmetric growth and institutions in an interdependent world, *Journal of Political Economy*, to appear.

[2] Amir, R. (1996a), Cournot oligopoly and the theory of supermodular games, *Games and Economic Behavior*, 15, 132-148.

[3] Amir (1996b), Sensitivity analysis of multisector optimal economic dynamics, *Journal of Mathematical Economics,* 25, 123-141.

[4] Amir, R. (2000). Modelling imperfectly appropriable R&D via spillovers. *International Journal of Industrial Organization*, 18, 1013-1032.

[5] Amir, R. (2005), Supermodularity and complementarity in economics: an elementary survey, *Southern Economic Journal,* 71, 636-660.

[6] Amir, R., Garcia F. and M. Knauff (2010). Symmetry-breaking in two-player games via strategic substitutes and diagonal nonconcavity: A synthesis, *Journal of Economic Theory* 145, 1968-1986.

[7] Amir, R. and V. Lambson (2000), On the effects of entry in Cournot markets, *Review of Economic Studies*, 67, 235-254.

[8] Amir, R., Wooders, J. (1999). Effects of one-way spillovers on market shares, industry price, welfare, and R&D cooperation. *Journal of Economics and Management Strategy,* 8, 223-249.

[9] Amir, R., Wooders, J. (2000). One-way spillovers, endogenous innovator/imitator roles, and research joint ventures. *Games and Economic Behavior,* 31, 1-25.

المنارة للاستشارات
www.manaraa.com

[10] Anderson, S. P., A. de Palma, and B. Kreider (2001), Tax incidence in differentiated product oligopoly, *Journal of Public Economics*, 81, 173-192.

[11] Bailey, J. P. (1998), Intermediation and electronic markets: aggregation and pricing in Internet commerce, Ph.D. Thesis, Technology, Management and Policy, M.I.T.

[12] Basu, K. and T. Cordella (2016), Asymmetric punishment as an instrument of corruption control, *Journal of Public Economic Theory*, 18, 831–856.

[13] Bergstrom, T. and Varian, H. (1985), Two remarks on Cournot equilibria. *Economics Letters*, 19, 5-8.

[14] Bernstein, J. and I. Nadiri (1988). Interindustry R&D spillovers, rates of return, and production in high-tech industries. *American Economic Review*, 78, 429-34.

[15] Bester, H. and E. Petrakis (1995), Price competition and advertising in oligopoly, *European Economic Review,* 39, 1075-1088.

[16] Billand, P., C. Bravard, S. Chakrabarti and S. Sarangi (2016). Business intelligence and multimarket competition, *Journal of Public Economic Theory*, 18, 248–267.

[17] Bloch, F. and P. Belleflamme (2004), Market sharing agreements and collusive networks, *International Economic Review*, 45, 387-411.

[18] Boone, J. and J. Potters (2002), Transparency, prices and welfare with imperfect substitutes, *Economics Letters*, 93, 398-404.

[19] Brander, J.A. and B.J. Spencer (1985). International R&D rivalry and industrial

المنارة للاستشارات
www.manaraa.com

strategy. *The Review of Economic Studies* 50, 707-722.

[20] Brynjolfsson, E. and M. D. Smith (2000), Frictionless commerce? A comparison of Internet and conventional retailers, *Management Science*, 46(4), 563-585.

[21] Bulow, J., Huang, M., and Klemperer, P. (1999). Toeholds and Takeovers. Journal of Political Economy, 107, 427-454.

[22] Burdett, K. and K.L. Judd (1983), Equilibrium price dispersion, *Econometrica,* 51, 955-969.

[23] Burr, C., Knauff, M., Stepanova, A. (2013). On the prisoner's dilemma in R&D with input spillovers and incentives for R&D cooperation. *Mathematical Social Sciences*, 66, 254–261.

[24] Chalioti, E. (2015). Incentive contracts under product market competition and R&D spillovers. *Economic Theory*, 58, 305-328.

[25] Chatterjee, A. (2017). Endogenous comparative advantage, gains from trade and symmetry-breaking, *Journal of International Economics,* to appear.

[26] Cho, I. K., Jewell, K., & Vohra, R. (2002). A simple model of coalitional bidding. *Economic Theory*, 19(3), 435-457.

[27] d'Aspremont, C., Jacquemin, A., Gabszewicz, J. J., and Weymark, J. A. (1983). On the stability of collusive price leadership. *Canadian Journal of economics*, 17-25.

[28] d'Aspremont, C., Jacquemin, A. (1988). Cooperative and noncooperative R&D in duopoly with spillovers. *American Economic Review,* 78, 1133–1137.

[29] d'Aspremont, C., Jacquemin, A. (1990). Erratum. *American Economic Review,*

المنارة للاستشارات

www.manaraa.com

80, 641–642.

[30] Daughety, A. (1990), Beneficial concentration, *American Economic Review,* 80, 1231-37.

[31] DeBrock, L. M., & Smith, J. L. (1983). Joint bidding, information pooling, and the performance of petroleum lease auctions. *The Bell Journal of Economics*, 395-404.

[32] Dubey, P., O Haimanko, A Zapechelnyuk (2006). Strategic complements and substitutes, and potential games, *Games and Economic Behavior*, 54, 77-94.

[33] Echenique, F. (2002). Comparative statics by adaptive dynamics and the correspondence principle, *Econometrica*, 70, 833-844.

[34] Edlin, A. and C. Shannon (1998), Strict monotonicity in comparative statics, *Journal of Economic Theory*, 81, 201-219.

[35] Ferrier, G. D., J. Reyes and Z. Zhu (2016). Technology diffusion on the international trade network, *Journal of Public Economic Theory*, 18, 291–312.

[36] Frank, C. (1965), Entry in a Cournot market, *Review of Economic Studies*, 32, 245-50.

[37] Gaudet, G. and S. Salant (1991), Uniqueness of Cournot equilibrium: new results from old methods, *Review of Economic Studies*, 58, 399-404.

[38] Gorbenko, A. and Malenko, A. (2014). Strategic and Financial Bidders in Takeover Auctions. The Journal of Finance, 69, 2513-2555.

[39] Griliches, Z. (1995). R&D and productivity: Econometric results and productiv-

المنارة للاستشارات
www.manaraa.com

ity issues, in P. Stoneman ed., *Handbook of the Economics of Innovation and Technological Change*, Basil Blackwell, Oxford.

[40] Hahn, F. (1962), The stability of the Cournot oligopoly Solution, *Review of Economic Studies,* 29, 329-221.

[41] Hendricks, K., and Porter, R. H. (1992). Joint Bidding in Federal OCS Auctions. *The American Economic Review*, 82, 506-511.

[42] Hinloopen, J. (1997). Subsidizing cooperative and noncooperative R&D in duopoly with spillovers, *Journal of Economics,* 66, 151-175.

[43] Hinloopen, J. (2000). Strategic R&D co-operatives, *Research in Economics,* 54, 153-185.

[44] Hoernig, S. (2003), Existence of equilibrium and comparative statics in differentiated goods Cournot oligopolies, *International Journal of Industrial Organization*, 21, 989-1020.

[45] Holt, C. (1995), Industrial Organization: A Survey of the Results of Laboratory Experiments, in *Handbook of Experimental Economics*, A. Roth and J. Kagel eds., Princeton University Press.

[46] Ireland, J. N. (1993), The provision of information in a Bertrand oligopoly, *Journal of Industrial Economics*, 41, 61-76.

[47] Jin, J.Y., Troege, M. (2006). R&D competition and endogenous spillovers. *The Manchester School*, 74, 40-51.

[48] Kamien, M., Muller, E., and Zang, I. (1992). Research joint ventures and R&D

المنارة للاستشارات

www.manaraa.com

cartels. *American Economic Review,* 82, 1293-1306.

[49] Katz, M. (1986). An analysis of cooperative research and development. *RAND Journal of Economics,* 17, 527-543.

[50] Kohlberg, E. and J.-F. Mertens (1986), Strategic Stability of Equilibria, *Econometrica*, 54, 1003-37.

[51] Koutsougeras, L. (2009), Convergence of Strategic Behavior to Price Taking, *Games and Economic Behavior,* 65, 234-241.

[52] Krishna, V., & Morgan, J. (1997). (Anti-) Competitive Effects of Joint Bidding and Bidder Restrictions. Woodrow Wilson School, Princeton University.

[53] Lee, H. G., Westland, C. and Hong, S. (2000), Impacts of electronic marketplaces on product prices: an empirical study of AUCNET case, *International Journal of Electronic Commerce*, 4(2), 45-60.

[54] MacManus, M. (1962), Number and Size in Cournot Equilibrium, *Yorkshire Bulletin of Economic and Social Research*, 14, 14-22.

[55] Mankiw, N.G. and Whinston, M. D. (1986), Free Entry and Social Inefficiency, *Rand Journal of Economics*, 17, 48-58.

[56] Mares, V., and Shor, M. (2012). On the competitive effects of bidding syndicates. *The BE Journal of Economic Analysis & Policy,* 12(1).

[57] Marshall, R., and Marx, L. (2009). The Vulnerability of Auctions to Bidder Collusion, *The Quarterly Journal of Economics*, 124, 883–910.

[58] Martin, S. (2002). Spillovers, appropriability and R&D. *Journal of Economics,*

المنارة للاستشارات
www.manaraa.com

75, 1-22.

[59] Matsuyama, K. (2002). Explaining diversity: symmetry-breaking in complementarity games, *American Economic Review*, 92, 241-246.

[60] McAfee, R. P., and McMillan, J. (1992). Bidding rings. *The American Economic Review*, 579-599.

[61] McDonald, S., Poyago-Theotoky, J. (2017). Green technology and optimal emissions taxation. *Journal of Public Economic Theory*, to appear.

[62] de Meza, D. (1985), A Stable Cournot-Nash Industry Need not be Quasi-Competitive, *Bulletin of Economic Research*, 37, 153-156.

[63] Milgrom, P. and J. Roberts (1990), Rationalizability, learning, and equilibrium in games with strategic complementarities, *Econometrica,* 58, 1255-78.

[64] Milgrom, P. and J. Roberts (1994), Comparing equilibria, *American Economic Review*, 84, 441-459.

[65] Milgrom, P. and C. Shannon (1994), Monotone comparative statics, *Econometrica*, 62, 157-80.

[66] Mollgaard, H.P. and P.B. Overgaard (2001), Market transparency and competition policy, Copenhagen Business School.

[67] Monaco, A. and T. Sabarwal (2016), Games with strategic complements and substitutes, *Economic Theory*, 62 (1-2), 65-91.

[68] Monderer, D. and L. Shapley (1996), Potential games, *Games and Economic Behavior*, 14, 124-143.

المنارة للاستشارات

www.manaraa.com

[69] Neyman, A. (1997), Correlated equilibrium and potential games, *International Journal of Game Theory*, 26, 223-227.

[70] Nilsson, A. (1999), Transparency and competition, mimeo., Stockholm School of Economics.

[71] Novshek, W. (1980), Cournot Equilibrium with free entry, *Review of Economic Studies*, 47, 473-486.

[72] Novshek, W. (1985), On the Existence of Cournot Equilibrium, *Review of Economic Studies*, L II, 85-98.

[73] Okuguchi, K. (1973), Quasi-competitiveness and Cournot oligopoly, *Review of Economic Studies*, 40, 145-148.

[74] Quah, J.K.-H. and B. Strulovici (2009). Comparative statics, informativeness, and the interval dominance order. *Econometrica*, 77, 1949–1992.

[75] Roberts, J. and H. Sonnenschein (1976), On the existence of Cournot equilibrium without concave profit functions, *Journal of Economic Theory*, 13, 112-117.

[76] Rockafeller, T. (1970), *Convex Analysis*, Princeton University Press, Princeton, N.J.

[77] Roy, S. and T. Sabarwal (2012). Characterizing stability properties in games with strategic substitutes, *Games and Economic Behavior*, 75, 337-353.

[78] Ruff, L. (1969). Research and technological progress in a Cournot economy. *Journal of Economic Theory*, 1, 397-415.

[79] Ruffin, R. J. (1971), Cournot oligopoly and competitive behavior, *Review of*

المنارة للاستشارات

www.manaraa.com

Economic Studies, 38, 493-502.

[80] Salant, S. W., Shaffer, G. (1998). Optimal asymmetric strategies in research joint ventures. *International Journal of Industrial Organization*, 16, 195-208.

[81] Salant, S. W., Shaffer, G. (1999). Unequal treatment of identical agents in Cournot equilibrium. *American Economic Review*, 89, 585-604.

[82] Schultz, C. (2004), Market transparency and product differentiation, *Economics Letters*, 83, 173-178.

[83] Schultz, C. (2005), Transparency on the consumer side and tacit collusion, *European Economic Review,* 49, 279-97.

[84] Seade, J. (1980), On the effects of entry, *Econometrica*, 48, 479-489.

[85] Singh, N. and X. Vives (1984), Price and quantity competition in a differentiated duopoly, *Rand Journal of Economics*, 15, 546-554.

[86] Singh, R. (2015) Takeover Bidding with Toeholds: The Case of the Owner's Curse. *The Review of Financial Studies*, 11, Pages 679–704.

[87] Soubeyran, A. and N. Van Long (1999). Asymmetric contributions to research joint ventures. *The Japanese Economic Review*, *50*, 122-137.

[88] Spence, M. (1984). Cost reduction, competition, and industry performance. *Econometrica*, 52, 101-121.

[89] Stepanova, A., Tesoriere (2011). R&D with spillovers: monopoly versus noncooperative and cooperative duopoly, *The Manchester School* 79, 125-144.

[90] Stahl, D.O. (1989), Oligopolistic pricing with sequential consumer search, *Amer-*

المنارة للاستشارات

www.manaraa.com

ican Economic Review, 79, 700-712.

[91] Suzumura, K. (1992). Cooperative and noncooperative R&D in an oligopoly with spillovers, *The American Economic Review*, 1307-1320.

[92] Tesoriere, A. (2008). Endogenous R&D symmetry in linear duopoly with one-way spillovers. *Journal of Economic Behavior & Organization*, 66, 213-225.

[93] Topkis, D. (1978), Minimizing a Submodular Function on a Lattice, *Operations Research*, 26, 305-321.

[94] Topkis, D. (1979), Equilibrium Points in Nonzero-Sum n-Person Submodular Games, *SIAM Journal of Control and Optimization*, 17, 773-787.

[95] Topkis, D. (1998), *Supermodularity and Complementarity.* Princeton University Press.

[96] Troyan, P. (2017). Collusion and signaling in auctions with interdependent values. *Journal of Economic Theory*, 170, 319-345.

[97] Varian, H. (1980), A model of sales, *American Economic Review,* 70, 651-659.

[98] Vives, X. (1990), Nash equilibrium with strategic complementarities, *Journal of Mathematical Economics*, 19, 305-321.

[99] Vives, X. (1999), Oligopoly pricing: old ideas and new tools, *MIT Press*, Cambridge, MA.

[100] Yazici, H. (2016). Optimal subsidization of business Start-ups, *Journal of Public Economic Theory* 18, 589–609.

المنارة للاستشارات

www.manaraa.com